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Pamphlet, '*A Supplement to a tract entitled 'A treatise on the construction and properties of arches' published in the year 1801*'

Date:

1804

Description:

Pamphlet by George Atwood, Fellow of the Royal Society, entitled A Supplement to a tract entitled 'A treatise on the construction and properties of arches' published in the year 1801 (London, 1804), with diagrams and tables of data relating to the forces operating within arches, 1804.

1 pamphlet

A

SUPPLEMENT TO A TRACT,

ENTITLED

A TREATISE ON THE CONSTRUCTION AND PROPERTIES OF ARCHES,

PUBLISHED IN THE YEAR 1801;

AND CONTAINING

PROPOSITIONS FOR DETERMINING THE WEIGHTS OF THE SEVERAL SECTIONS WHICH CONSTITUTE AN ARCH,

INFERRED FROM THE ANGLES.

ALSO CONTAINING A

DEMONSTRATION OF THE ANGLES OF THE SEVERAL SECTIONS, WHEN THEY ARE INFERRED FROM THE WEIGHTS THEREOF.

TO WHICH IS ADDED,

A DESCRIPTION OF ORIGINAL EXPERIMENTS TO VERIFY AND ILLUSTRATE THE PRINCIPLES IN THIS TREATISE.

WITH

OCCASIONAL REMARKS ON THE CONSTRUCTION OF AN IRON BRIDGE OF ONE ARCH, PROPOSED TO BE ERECTED OVER THE RIVER THAMES AT LONDON.

PART II.

BY THE AUTHOR OF THE FIRST PART.

LONDON:

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1804.

PREFACE.

A PLAN for constructing an Iron Bridge of one arch, to be erected over the River Thames, designed by Messrs. Telford and Douglass, and proposed to the Committee of the House of Commons for the further Improvement of the Port of London, has excited considerable attention, both from the novelty and magnitude of the design, and the evident advantages to navigation which would attend such a structure; yet as some doubts arose respecting the practicability of erecting such an edifice, and the prudence of attempting it, the Committee judged it necessary for their own information, as well as to furnish the House with some grounds by which an opinion might be formed, to propose the following Queries, which were therefore transmitted, together with the engraved designs of Messrs. Telford and Douglass, and the explanatory drawings annexed, to such persons as were supposed to be most capable of affording them information.

The following are the Queries that were drawn up and transmitted to the Persons whose Names are undermentioned. (See Page vi.)

QUERIES.

I. What parts of the arch are to be considered as wedges, which act on each other by gravity and pressure, and what part merely as weight, acting by its gravity only, similar to the walls and other loading commonly erected on the arches of stone bridges; or does the whole act as one frame of iron, which cannot be destroyed but by crushing its parts?

Query II. Whether the strength of the arch is affected, and in what manner, by the proposed increase of its width towards the two extremities or abutments, when considered both vertically and horizontally; and if so, what form should the bridge gradually acquire?

III. In what proportion should the weight be distributed, from the centre to the abutments, to make the arch uniformly strong?

IV. What pressure will each part of the bridge receive, supposing it divided into any given number of equal sections, the weight of the middle section being known; and on what part, and with what force, will the whole act upon the abutments?

V. What additional weight will the whole bridge sustain, and what will be the effect of a given weight placed on any of the fore-mentioned sections?

VI. Supposing the bridge executed in the best manner, what horizontal force will it require, when applied to any particular part, to overturn it, or press it out of the vertical position?

VII. Supposing the span of the arch to remain the same, and to spring ten feet lower, what additional strength would it give to the bridge; or, making the strength the same, what saving may be made in the materials; or, if instead of a circular arch, as in the Print and Drawings, the bridge should be made in the form of an elliptical arch, what would be the difference in effect as to strength, duration, and expense?

VIII. Is it necessary or adviseable to have a model made of the proposed bridge, or any part of it, of cast iron; if so, what are the objects to which the experiments should be directed, to the equilibration only, or to the cohesion of the several parts, or to both united, as they will occur in the iron work of the intended bridge?

IX. Of what size ought this model to be made, and in what relative proportion will experiments on the model bear to the bridge when executed?

X. By what means may ships be best directed in the middle stream, or prevented from driving to the side, and striking the arch; and what is the probable consequence of such a stroke?

XI. The weight and lateral pressure of the bridge being given, can abutments be made in the proposed situation, for London Bridge to resist that pressure?

Query XII. The weight of the whole iron work being given, can a centre or scaffolding be erected over the river, sufficient to carry the arch, without obstructing those vessels which at present navigate that part?

XIII. Whether would it be most adviseable to make the bridge of cast and wrought iron combined, or of cast iron only; and if of the latter, whether of the hard and white metal, or of soft grey metal, or of gun metal?

XIV. Of what dimensions ought the several members of the iron work to be made, to give the bridge sufficient strength?

XV. Can frames of iron be made sufficiently correct to compose an arch of the form and dimensions as shewn in the Drawings No. 1 and 2, so as to take an equal bearing in one frame, the several parts being connected by diagonal braces, and joined by iron cement, or other substance? N. B. The Plate XXIV. in the Supplement to the Third Report, is considered as No. 1.

XVI. Instead of casting the ribs in frames of considerable length and breadth, as shewn in the Drawings No. 1 and 2, would it be more adviseable to cast each member of the ribs in separate pieces of considerable length, connecting them together with diagonal braces, both horizontally and vertically, as in No. 3.?

XVII. Can an iron cement be made that will become hard and durable; or could liquid iron be poured into the joints?

XVIII. Would lead be better to use in the whole, or any part, of the joints?

XIX. Can any improvements be made upon the Plans, so as to render the bridge more substantial and durable, and less expensive; if so, what are those improvements?

XX. Upon considering the whole circumstance of the case, agreeably to the Resolutions of the Select Committee, as stated at the conclusion of their Third Report,* is it your opinion, that an arch of 600 feet span,

* The Resolutions here referred to are as follow:

That it is the opinion of this Committee, that it is essential to the improvement and accommodation of the Port of London, that London Bridge should be rebuilt, on such a construction as to permit a free passage, at all times of the tide, for ships as expressed in the Drawings produced by Messrs. Telford and Douglass, on the same plane, with any improvements you may be so good as to point out, is practicable and adviseable, and capable of being rendered a durable edifice?

Query XXI. Does the Estimate communicated herewith, according to your judgment, greatly exceed, or fall short of, the probable expence of executing the Plan proposed, specifying the general grounds of your opinion?

After paying every attention to the subject which the importance of it demanded, it appeared for many reasons absolutely necessary, for furnishing satisfactory answers to the above Queries, to investigate the properties of arches from their first principles. The substance of these properties is comprised in a Tract, entitled a Dissertation on the Construction and Properties of Arches, published in the year 1801, and continued in the present Treatise, now offered to the Public as a Supplement to the former Tract. The

of such a tonnage, at least, as the depth of the river would admit at present, between London Bridge and Blackfriars Bridge.

That it is the opinion of this Committee, that an iron bridge, having its centre arch not less than 65 feet high in the clear above high-water mark, will answer the intended purposes, with the greatest convenience, and at the least expense.

That it is the opinion of this Committee, that the most convenient situation for the new bridge will be immediately above St. Saviour's Church, and upon a line leading from thence to the Royal Exchange.*

ANSWERS BY

1. Dr. Maskelyne,
2. Professor Robertson,
3. Professor Playfair,
4. Professor Robeson,
5. Dr. Milner,
6. Dr. Hutton,
7. Mr. Atwood,
8. Colonel Twiss,
9. Mr. Jessop,
10. Mr. Rennie,
11. M. Watt,
12. Mr. Southern,
13. Mr. Reynolds,
14. Mr. Wilkinson,
15. Mr. Bage,
16. General Bentham,
17. Mr. Wilson.

* See the Report from the Select Committee upon the Improvement of the Port of London.

reader will perceive that most of the propositions in these Dissertations are entirely new, and that they have been verified and confirmed, by new and satisfactory experiments, on Models constructed in brass by Mr. Berge of Piccadilly, whose skill and exactness in executing works of this sort are well known to the Public. Considering the importance of the subject, and the diversity of opinions which has prevailed respecting the construction of arches, and the principles, on which they are founded, it seems requisite, that the final determination of the plan for erecting the bridge of one arch in question, should be subjected to a rigorous examination, in order to discover if any, and what, errors might be found in them. The best means of effecting this appears to be by a publication, in which the propositions recommended for adoption being fairly stated, every person, who is of a different opinion, may have an opportunity of explaining his ideas on the subject, and of suggesting any different modes of construction, that are judged to be less liable to objection. To persons interested in these inquiries, it may be satisfactory to be informed, that the properties of arches, which are comprised in this latter Tract, have been found, on a careful and minute examination, and comparison, in no instance inconsistent with those, which are the subject of investigation in Part the First, but rather appear to strengthen and confirm the theory before published, allowing for the differences in the initial force or pressure, expressed in page 2, and in Figs. 1 and 2, inserted in this Tract, representing the different dispositions of the key-stones, from whence conclusions arise very different from each other, although all of them are strictly consistent with the laws of geometry and statics. It is particularly observable, that the deductions of the weights and pressures arising from a supposition of a single key-stone, do not exhibit conclusions which are strictly true, but require the addition or subtraction of certain differences* to make them consistent: whereas on the more correct supposition of two key-stones, corresponding with the case, in which the initial pressure is in a direction parallel to the horizon, the conclusions derived from this principle are geometrically true, requiring no correction or alteration whatever; being in themselves certain and unalterable propositions. Practical inferences may be deduced from adopting either the principle of a single key-stone, or the more correct one of two equal key-stones, the differences, which are the consequences, whether subtractive, or additive, being so extremely minute as not to be made sensible in practice. With respect to the principal object of these inquiries, those which are expressed in the 19th and 20th Queries, deserve particular attention.

" Can any improvements be made upon the Plans, so as to ren-
" der the bridge more substantial and durable, and less expensive;
" if so, what are those improvements?"

" Upon considering the whole circumstance of the case, agree-
" ably to the resolution of the Select Committee, as stated at the
" conclusion of their Third Report, is it your opinion that an arch of
" 600 feet span, as expressed in the drawings produced by Messrs.
" Telford and Douglass, on the same plan, with any improvement
" you may be so good as to point out, is practicable and adviseable,
" and capable of being a durable edifice?"

It seems probable that the best plan of rendering the bridge a substantial and durable edifice, would be by making the weights of the several sections, such as those plans which are numerically expressed in Tables No. V, VI, VIII, IX, annexed to Part II. in this Treatise, and their pressures on the abutments, balance each

* See Table VI. at the end of Part the First.

other, so that the whole building, when erected, may have a disposition to remain at rest; which will be the property of all the structures of arches, which are numerically expressed in the Tables, that are subjoined both in the First and Second Part of this Treatise. In some of the plans, particularly those which are drawn in a circular form, inconveniences arise from the figure thereof, that render them unfit to be adopted for the purpose of erecting a bridge: to obviate this difficulty it might be advisable, that the curve of the arch should not be formed of a circular or other specific figure, but that the line coinciding with the road-way might be either rectilinear, or a curve not greatly deviating from a right line, so that if the bridge should be constructed according to any of the plans pointed out in the preceding pages, the advantages therein proposed, would be realized, without the inconveniences arising from a circular form.

It may be considered as a matter of surprise, that on a subject so truly experimental as the construction of arches appears to be, so very few accounts of original experiments on the subject are to be found, in the philosophical transactions of this or other countries of Europe, or in the literary publications which have appeared in the world during the last and preceding centuries; it possibly may be objected against placing any reliance on experiments of this sort, that they are formed on a supposition, that no impediment is caused by friction, cohesion, and tenacity of the surfaces in contact; whereas in reality those powers operate in preventing the surfaces from freely sliding over each other, and consequently an adequate allowance ought to be made on this account in forming inferences from these experiments: but it seems certain, that in whatever degree friction, and the other impediments to motion, may act on the models, it is by rendering the whole structure more secure from disunion. The effects therefore of similar or other impediments, such as may be supposed to take place in the construction of real bridges, will have a much greater effect when they consist of iron braces and fastenings of various kinds; by which all efforts to disunite the sections are immediately counteracted.

The effects of this will be not only to prevent the separation of the sections by any casual force, tending to disunite them, but will likewise secure the edifice from the more silent, but not less destructive assaults of time: for when the sections of an arch are not duly balanced, every heavy weight which passes over the road-way, even the motion of a lighter carriage, must create a tendency to separate the sections by degrees, and at length entirely to disunite them; an evil to be remedied only by a requisite equilibration of parts of the bridge.

On a review of the whole, whether the subject is considered theoretically, as depending on the laws of motion, or practically, on the construction of models erected in strict conformity to the theory, it would seem difficult to suppose, that any principle for erecting a bridge of one arch would be adopted, that is very different from those, that have been the subject of the preceding pages: nevertheless, as the most specious theories have been known to fail, when applied to practice, in consequence of very minute alterations in the conditions; and as it is scarcely possible to frame experiments adequate to the magnitude of the intended structure, the Author of this Treatise thinks it incumbent upon him to state freely the doubts which remain upon his mind, respecting the construction of the bridge intended; suggesting, at the same time, such ideas, as have occurred to him, which probably may contribute to remove or to explain those doubts; particularly by causing an arch to be erected, the span of which is from 20 to 50 feet, the expense of which would be of little moment in the case of its success; and, on a supposition, that the experiment should fail, the important consequences that would probably arise from the observation of such a fact would, in the opinion of many persons, amply compensate for its failure. A doubt occurred during the construction of the flat arch,* whether the angles at the summit were most conveniently fixed at 2° 38′ 0″, or whether those angles should not subtend 5°, 10°, 15°, or any other angles, which might better contribute to the strength and stability of the entire structure. Since the materials, of which the Models are formed, are of a soft and elastic nature, which yields in some degree to the force of pressure; this circumstance, joined to that of making the angle subtended at the centre of the circle no greater than 2° 38′ 0″, prevents these sections from having much hold on the contiguous sections above them, and creates some difficulty and attention in adjusting the Model No. 2, to an horizontal plane, suggesting the necessity of forming the angles of the first or highest sections at 5°, or some greater angle, by which the holdings would be more effectually secured; but it is to be remembered, that this source of imperfection could not exist if the sections were made of materials perfectly hard and unelastic; and the Model having been constructed as an experiment, it seems proper that the angles of the first sections should be formed on the smallest allowable dimensions, in order to observe more distinctly the advantages which would arise from making the angles larger in any subsequent experiment, if any should be approved of, previously to a final determination of the plan to be adopted for erecting the iron bridge. It is to be

* The Model No. 2, so called to distinguish it from the Model No. 1, in the form of a semicircular arch.

observed, that no imperfection of the kind which is here spoken of, takes place in the Model of the arch No. 2, after it has been carefully erected: but a larger angle seems to be preferable for the angles of the first sections, from the difficulty which subsists, at present, in adjusting the Model of the arch No. 2, to the true horizontal plane, so much exceeding the trouble and attention in adjusting the Model No. 1.

Many thanks and acknowledgments are due to Mr. Telford and several other engineers, who have had the goodness to favour the Author with their able advice and assistance, in answering such questions as he had occasion to propose to them, respecting the original plan of this Treatise, and subsequently concerning the practical experiments, accounts of which are contained in it.

G. A.

London,
29th November, 1803.

A

DISSERTATION

ON THE

CONSTRUCTION AND PROPERTIES OF ARCHES.

PART II.

THE sections or portions of wedges which constitute an arch may be disposed according to two several methods of construction, which are represented by Fig. 1 and Fig. 2. In Fig. 1 the highest section, or key-stone, is bissected by the vertical plane VO, which divides the entire arch into two parts, similar and equal to each other. In Fig. 2, *two* highest sections A, A, similar and equal to each other, are placed contiguous and in contact with the vertical line VO. The former plan of construction has been before the subject of investigation, in a tract on arches, and published in the year 1801. It remains to consider the properties which result from disposing the sections according to the last-mentioned plan in Fig. 2.

The first material circumstance which occurs is the difference in the direction of the initial pressure, which in the former case, Fig. 1, was inclined to the horizon in the direction EQ perpendicular to AB; whereas, according to the latter disposition, Fig. 2, of the key-stone, the initial pressure is parallel to the horizon in

B

the direction QR. In any arch of equilibration in which two equal and similar sections occupy the summit of the arch, the initial force or pressure is parallel to the horizon, and is to the weight of the first section as radius is to the tangent of the angle of that section.

For let the two highest equal sections, A, A, be represented by Fig. 3, when they form a portion of an arch of this description; let V*v* T*a* represent one of these equal highest sections. Through any point Q, of the line VO, draw QP perpendicular to the line TO, QR parallel, and PR perpendicular, to the horizon; then will the three forces, by which the wedge A is supported in equilibrio, be represented in quantity and direction, by the lines QP, QR, and PR; of which, QP denotes the pressure between the surface TO and the surface of the section B, which is contiguous to it. QR is the force which acts in a direction parallel to the horizon, and is counterbalanced by the reaction of the other section A, similar and equal to the former: and PR measures the weight of the section A. Because PQR is a right-angled triangle, the following proportion will be derived from it: as the horizontal force QR is to the weight of the section A, or PR, so is radius or QR to PR. The tangent of the angle PQR = VOT, which being equal to the angle contained by the sides V*v*, *a*T of the wedge A, may be denoted by A°: finally, if the weight of the section A be put equal to w, we shall have the horizontal force at the summit of the arch $= \frac{w}{\text{tang. } A^\circ} = w \times \text{cotang. } A^\circ$, radius being = 1; from this determination the following construction is derived: having given the several angles of the sections A°, B°, C°, D°, together with the weight of the first section A, to ascertain by geometrical construction, the weights of the successive sections B, C, D, &c. when the arch is balanced in equilibrio. AΔ, AB, BC, CD, &c.

represent the bases of the sections in Fig. 4: through the points A, B, C, D, &c. draw the indefinite lines A*a*, B*b*, C*c*, D*d*, &c. perpendicular to the horizon; through any point X, in the line AF, draw the indefinite line XZ parallel to the horizon; let A*a* denote the weight of the section A; and through the point *a* draw *az*, at right angles to AF; and in the line XZ take a part XM, which shall be to the line A*a* as radius is to the tangent of the angle VOF or A°; so shall XM represent, in quantity and direction, the pressure between the first section A and the vertical plane VO; or, when both semiarches are completed, the line XM will represent the pressure between the contiguous vertical surfaces of the two highest sections A, A. Through the point M draw MRV perpendicular to OF; and in this line produced, take MN = to *za*; and make QV = to RN, which will be to radius, as radius is to the sine of VOA or A°. For because the line XM is to A*a* as radius is to the tangent of VOA or A°; if the sin. of A° be put = *s*, and the cos. A° = *c* to radius 1, this will give $RM = \frac{Aa \times c^2}{s}$; and because $MN = za = Aa \times s$, and RM $= \frac{Aa \times c^2}{s}$; RM + MN or $RN = \frac{Aa \times \overline{s^2 + c^2}}{s} = VQ = \frac{Aa}{s}$, which quantity is to radius, as radius is to the sin. of VOA or A°: and VQ or $RN = \frac{Aa}{s}$ is the measure of the entire pressure on the abutment OF.

To construct the weight of the section B, and the pressure on the next abutment OG, through the point Q, draw KT perpendicular to GS, and from any point B, in the line BG, set off B*z* = to VS: through the point *z* draw *zb* perpendicular to G*b* cutting off B*b*, which will be equal to the measure of the weight of the section B; from the point Q in the line KT produced set off QT = to

B 2

zb: also in the line KT, make $K\Sigma =$ to ST, then $K\Sigma$ or ST is the measure of the pressure on the abutment OG of the section C. On the same principles, the weights of the sections C and D, as well as of the sections following, are geometrically constructed, Cz being set off $=$ WK, and $Dz =$ to ΠI; from this construction, when completed, the general expressions for the weights of the sections are inferred, which are inserted in the 13th and 14th pages in the former tract, except that the initial pressure, arising from a different disposition of the key-stone, represented in Figs. 1 and 2, in consequence of which the initial pressure is $p' = w \times \text{cotang. } A^{\circ}$, instead of $p = \frac{w}{2 \times \text{sin. } \frac{1}{2} A^{\circ}}$ in the former tract.

In this manner the weights of the several sections and pressures on the abutments, are found to be as underneath.

Sections.	Weights of the sections on the vertical abutments.	Pressures.
		$p' = w \times \text{cotang. } A^{\circ}$
	$A = w$	$p = p' \times \text{cos. } A^{\circ} + p' \times \text{sin. } A^{\circ} \times \text{tang. } V^{a}$
	$B = p \times \text{sin. } B^{\circ} \times \text{sec. } V^{b}$	$q = p \times \text{cos. } B^{\circ} + p \times \text{sin. } B^{\circ} \times \text{tang. } V^{b}$
	$C = q \times \text{sin. } C^{\circ} \times \text{sec. } V^{c}$	$r = q \times \text{cos. } C^{\circ} + q \times \text{sin. } C^{\circ} \times \text{tang. } V^{c}$
	$D = r \times \text{sin. } D^{\circ} \times \text{sec. } V^{d}$	$s = r \times \text{cos. } D^{\circ} + r \times \text{sin. } D^{\circ} \times \text{tang. } V^{d}$
	$E = s \times \text{sin. } E^{\circ} \times \text{sec. } V^{e}$	$t = s \times \text{cos. } E^{\circ} + s \times \text{sin. } E^{\circ} \times \text{tang. } V^{e}$

When the angles of these sections are equal to each other, and consequently $A^{\circ} = B^{\circ} = C^{\circ} = D^{\circ}$; &c. in this case, the angles of the abutments will be as follows, $V^{a} = A^{\circ}$, $V^{b} = 2A^{\circ}$, $V^{c} = 3A^{\circ}$, and so on.

On these conditions, the weight of each individual section, as well as the pressures on the corresponding abutments, and the weights of the semiarches, may be inferred by the elementary rules of trigonometry, from the general expressions above inserted.

Weights of the sections, and the pressures on the corresponding abutments, when the angles of the sections are equal each, and $=$ to A°, sin. of each angle $= s$, cos. $= c$, radius $= 1$.

Weights of the sections.	Pressures on the lowest surface of each section.
	$p' = \frac{c}{s}$ - - - $=$ cotang. A°
$A = 1$	$p = \frac{c}{s} \times \frac{1}{c}$ . . $=$ cotang. A $\times$ sec. A
$B = \frac{1}{2c^2 - 1}$	$q = \frac{c}{s} \times \frac{1}{2c^2 - 1}$ . . $=$ cotang. A $\times$ sec. 2A
$C = \frac{1}{\overline{2c^2 - 1} \times \overline{4c^2 - 3}}$ . .	$r = \frac{c}{s} \times \frac{1}{4c^3 - 3c}$ $=$ cotang. A $\times$ sec. 3A
$D = \frac{1}{\overline{4c^2 - 3} \times \overline{8c^4 - 8c^2 + 1}}$	$s = \frac{c}{s} \times \frac{1}{8c^4 - 8c^2 + 1}$ $=$ cotang. A $\times$ sec. 4A
$E = \frac{1}{\overline{16c^4 - 20c^2 + 5} \times \overline{8c^4 - 8c^2 + 1}}$	$t = \frac{c}{s} \times \frac{1}{16c^5 - 20c^3 + 5c}$ $=$ cotang. A $\times$ sec. 5A

Sums of the weights of the sections, or weights of the semi-arches, when the angles of the sections are equal to each other, and $=$ to A°, sin. A° $= s$, cos. A° $= c$, radius $= 1$.

Sums of the weights.

$A = 1$ . . . . 1 . . $=$ cotang. A $\times$ tang. A

$A + B$ . . . . . . $= \frac{2c^2}{2c^2 - 1}$ $=$ cotang. A $\times$ tang. 2A

$A + B + C$ . . . $= \frac{4c^2 - 1}{4c^2 - 3}$ $=$ cotang. A $\times$ tang. 3A

$A + B + C + D$. $= \frac{8c^4 - 4c^2}{8c^4 - 8c^2 + 1}$ $=$ cotang. A $\times$ tang. 4A

$A + B + C + D + E = \frac{16c^4 - 12c^2 + 1}{16c^4 - 20c^2 + 5}$ $=$ cotang. A $\times$ tang. 5A.

When the angles of the sections, instead of being equal to each other, are of any given magnitude, the general demonstration of the weights of the sections, when adjusted to equilibration, and the corresponding pressures on the abutments, will require further examination of the principles on which the construction is formed; with the aid of such geometrical propositions as are applicable to the subject.

To consider, first, the pressures on the successive abutments which are, according to the construction, OV, OF, OG, OH, &c. it is to be proved, that the pressure on the vertical abutment OV = cotang. A°: the pressure on the abutment OF = cotang. A° × sec. A; the pressure on OG = cotang. A° × sec. $\overline{A° + B°}$; and the pressure on OH = cotang. A° × sec. $\overline{A° + B° + C°}$, and so on, according to the same law of progression; radius being = 1, the weight of the first section being also assumed = 1; if the weight of the first section should be any other quantity *w*, the pressures inferred must be multiplied by *w*.

The vertical line OV being parallel to the several lines A*a*, B*b*, C*c*, D*d*, &c. it appears that the angle *z*A*a* = FOV = A°, also *z*B*b* = GOV = A° + B°, *z*C*c* = HOV = A° + B° + C°, *z*D*d* = A° + B° + C° + D°, likewise the angle XMV = A°, VQK = B°, KΣI = C°, INΠ = D°, &c.

From these data the following determinations are obtained; the entire pressure QV on the abutment OF, consists of two parts, namely, RM = the wedge pressure; secondly, MN = *za*, which is that part of weight of the section A resting on the abutment FA, which is to the whole weight as *za* is to A*a*, or as the sine of the angle A° is to radius: the entire pressure therefore upon OF = RM + MN: but MR = MX × cos. A, and MN = *za* × tang. A to radius *z*A = sin. A: the pressure, therefore, on the line OF = $\frac{Aa \times \cos.^2 A}{\sin. A}$ + A*a* × sin. A = $\frac{Aa \times \overline{\cos.^2 A + \sin.^2 A}}{\sin. A}$ = $\frac{Aa}{\sin. A}$: but $\frac{Aa}{\sin. A}$ = cotang. A × sec. A; we have therefore arrived at the following determination: the entire pressure on the abutment OF = cotang. A° × sec. A, when the weight of the first section is assumed = 1.

The pressure on the abutment OG, that is ΣK, is to be proved $=$ cotang. A $\times$ sec. $\overline{A^\circ + B^\circ}$.

The pressure QV on the abutment preceding, or OF, has been shewn $=$ cotang. $A^\circ \times$ sec. A; but as the angle VQK $= B^\circ$, it follows that QS $=$ cotang. $A^\circ \times$ sec. $A^\circ \times$ cos. B°, and VS $=$ cotang. A $\times$ sec. $A^\circ \times$ sin. B°; but by the construction VS $= Bz$: therefore $Bz =$ cotang. $A^\circ \times$ sec. $A^\circ \times$ sin. B°: and because the angle $zBb = A^\circ + B^\circ$, zb is to Bz (cotang. $A^\circ \times$ sec. $A^\circ \times$ sin. B°) as tang. $A^\circ + B^\circ$ is to radius: the result is, that $zb =$ cotang. A $\times$ sec. A $\times$ sin. B $\times$ tang. $\overline{A^\circ + B^\circ}$: and since SQ $=$ cotang. $A^\circ \times$ sec. A $\times$ cos. B°; it follows that the entire pressure on OG $=$ SQ $+$ QT $=$ KQ $=$ cotang. $A^\circ \times$ sec. $A^\circ \times$ cos. $B^\circ +$ cotang. A $\times$ sec. $A^\circ \times$ sin. $B^\circ \times$ tang. $\overline{A^\circ + B^\circ}$. The subsequent geometrical proposition will verify this construction, and prove at the same time, the relation, in general, of the successive secants of the angles which are proportional to the entire pressures on the successive corresponding abutments.

Given any angle of an abutment A°, and the angle of the section B° next following, it is to be proved that sec. A° is to sec. $\overline{A + B}$ as 1 is to cos. B $+$ sin. B $\times$ tang. $\overline{A + B}$. That is, from the conditions given,

Sec. $A^\circ \times$ cos. B $+$ sec. $A^\circ \times$ sin. $B^\circ \times$ tang. $\overline{A^\circ + B^\circ} =$ sec. $\overline{A + B}$

From the elements of trigonometry, cos. B $+$ sin. B $\times$ tang. $\overline{A + B} =$ cos. B $+$ sin. B $\times \frac{\text{sin.}\,\overline{A+B}}{\text{cos.}\,\overline{A+B}} = \frac{\text{cos.}\,\overline{A+B} \times \text{cos.}\,B + \text{sin.}\,B \times \text{sin.}\,\overline{A+B}}{\text{cos.}\,\overline{A+B}}$ $= \frac{\text{cos.}\,\overline{A+B-B}}{\text{cos.}\,\overline{A+B}} = \frac{\text{cos.}\,A}{\text{cos.}\,\overline{A+B}}$: therefore cos. B $+$ sin. B $\times$ tang. $\overline{A+B}$ $\doteq \frac{\text{cos.}\,A}{\text{cos.}\,\overline{A+B}}$: multiply both sides by sec. A, the result will be: sec. A $\times$ cos. B $+$ sec. A $\times$ sin. B $\times$ tang. $\overline{A+B} = \frac{\text{cos.}\,A \times \text{sec.}\,A}{\text{cos.}\,\overline{A+B}} =$ sec. $\overline{A+D}$.

This proposition may be extended to ascertain, generally, the proportion of the successive secants in an arch of equilibration, by supposing an angle of an abutment M° to consist of the angles of several sections, such as A°, B°, C°, D°, $E^\circ = M^\circ$, if an additional section F° is next in order after E°; so that the whole arch may consist of sections, the sum of the angles of which $= M^\circ + F^\circ$, then it is to be proved that the secant of M°, is to the secant of $M^\circ + F^\circ$, as 1 to cos. F + sin. $F^\circ \times$ tang. $\overline{M^\circ + F^\circ}$, or sec. $M^\circ \times$ cos. M° + sec. $M^\circ \times$ sin. $F^\circ \times$ tang. $\overline{M^\circ + F^\circ} =$ sec. $\overline{M^\circ + F^\circ}$.

By the elements of trigonometry, cos. F + sin. $F \times$ tang. $\overline{M + F}$ $= \text{cos. } F + \text{sin. } F \times \frac{\text{sin. } \overline{M + F}}{\text{cos. } \overline{M + F}} = \frac{\text{cos. } F \times \text{cos. } \overline{M + F} + \text{sin. } F \times \text{sin. } \overline{M + F}}{\text{cos. } \overline{M + F}}$ or cos. F + sin. $F \times$ tang. $\overline{M + F} = \frac{\text{cos. } \overline{M + F - F}}{\text{cos. } \overline{M + F}} = \frac{\text{cos. } M}{\text{cos. } \overline{M + F}}$. Multiply both sides of the equation by sec. M, the result will be sec. $M \times$ cos. F + sec. $M \times$ sin. $F \times$ tang. $\overline{M + F} = \frac{\text{sec. } M \times \text{cos. } M}{\text{cos. } \overline{M + F}}$ $=$ sec. $\overline{M + F}$.

Thus the relation of the successive secants of the angles between the vertical line and the lowest surface of each section in any arch of equilibration is demonstrated, in general, and the measure of the pressures on the abutments proved to be equal to the weight of the first or highest section $\times$ cotang. $A^\circ \times$ sec. of the angle of that abutment: and, in general, any sec. of an angle of an abutment is shewn to be to the sec. of the angle of an abutment next following, in the proportion as 1 is to cos. of the angle of the section + sin. of the same angle $\times$ tang. of the sum of the angles from the summit of the arch to the abutment.

The ensuing geometrical proposition is intended to investigate the weights of the individual sections in an arch of equilibration:

also to infer the sums of the weights of the sections which form the respective semiarches. A, B, C, D, Fig. *6.* is a circular arc drawn from the centre O and with the distance OA. The arc $AB = A°$, $AC = B°$, $AD = C°$; AG is drawn a tangent to the circle at the point A; through the centre O and the points B, C, D draw the lines OBE, OCF, ODG: then the line AE will be a tangent to the arc AB, AF will be a tangent to the arc AC, and AG will be a tangent to the arc AD; through the points B, C, D draw the lines BH, CI, DK perpendicular to the line OA; then will BH be the sin. and OH the cos. of the arc $AB = A°$, CI and OI the sin. and cos. of the arc $AC = B°$ and $DK =$ the sin. and OK the cos. of the arc $AD = C°$ through C draw CM perpendicular to OE, so shall CM be the sin. of the arc CB.

The following proposition is to be proved: the difference of the tangents of the arcs AC and AB, or the line FE, is to the line CM, or the sine of the difference of the same arcs, so is 1 to the rectangle under the cosines of AB and AC, or $OH \times OI$: the demonstration follows, radius being $= 1$; the tangent of the arc $AB = \frac{\text{sin. } AB}{\text{cos. } AB}$, and tang. of the arc $AC = \frac{\text{sin. } AC}{\text{cos. } AC}$; therefore the difference of the tang. of AB and $AC = \frac{\text{sin. } AC}{\text{cos. } AC} - \frac{\text{sin. } AB}{\text{cos. } AB} = \frac{\text{sin. } AC \times \text{cos. } AB - \text{sin. } AB \times \text{cos. } AC}{\text{cos. } AB \times \text{cos. } AC}$; but the sin. of $AC \times$ cos. $AB -$ sin. $AB \times$ cos. $AC =$ sin. $\overline{AC - AB}$ $=$ the sin. of the difference of the same arcs $= CM$; therefore the difference of the tangents $EF = \frac{CM}{\text{cos. } AB \times \text{cos. } AC}$; which equation being resolved into an analogy, becomes the following proportion: as the difference of the tangents FE is to the sine of the difference of the arcs sin. $\overline{AC - AB}$, so is radius 1 to the rectangle under the cosines OI and OH, which is the proposition to be proved.

Since it has been shewn in the pages preceding, that the pressure on each abutment is $w \times$ cotang. $A^{\circ} \times$ sec. of the angle of that abutment, the pressures on the several sections will be expressed as follows:

Pressure on the vertical abutment VO $= w \times$ cotang. $A^{\circ} = w \times p'$ sec. V°.

	Pressures on the lowest surface of each section.
A	$p = w \times$ cotang. $A^{\circ} \times$ sec. V^{a}
B	$q = w \times$ cotang. $A^{\circ} \times$ sec. V^{b}
C	$r = w \times$ cotang. $A^{\circ} \times$ sec. V^{c}
D	$s = w \times$ cotang. $A^{\circ} \times$ sec. V^{d}
&c.	&c.

Let CB be an arc which measures the angle of any section, so that OF may represent the secant of the angle AOF, and OE $=$ the secant of the angle of the abutment AOE: the difference of the tangents FE $= \frac{CM}{\text{cos. AB} \times \text{cos. AC}} =$ sin. $B^{\circ} \times$ sec. of the angle AOB, $\times$ sec. of the angle AOC, or, according to the notation which has been adopted, the difference of the tangents FE $=$ sin. $B^{\circ} \times$ sec. of $V^{a} \times$ sec. V^{b}, radius being $= 1$.

The weight of the section B, by page 6, $= p \times$ sin. $B^{\circ} \times$ sec. V^{b}, but by the table in page above inserted, $p = w \times$ cotang. $A^{\circ} \times$ sec. wherefore the weight of the section B $= w \times$ cotang. $A^{\circ} \times$ sin. $B^{\circ} \times$ sec. $V^{a} \times$ sec. V^{b}: on the same principles the weights of the several sections will be expressed as underneath.

Sections.	Weights.
A	$= w \times$ cotang. $A^{\circ} \times$ sin. $A^{\circ} \times$ sec. $V^{\circ} \times$ sec. V^{a}
B	$= w \times$ cotang. $A^{\circ} \times$ sin. $B^{\circ} \times$ sec. $V^{a} \times$ sec. V^{b}
C	$= w \times$ cotang. $A^{\circ} \times$ sin. $C^{\circ} \times$ sec. $V^{b} \times$ sec. V^{c}
D	$= w \times$ cotang. $A^{\circ} \times$ sin. $D^{\circ} \times$ sec. $V^{c} \times$ sec. V^{d}
E	$= w \times$ cotang. $A^{\circ} \times$ sin. $E^{\circ} \times$ sec. $V^{d} \times$ sec. V^{e}
F	$= w \times$ cotang. $A^{\circ} \times$ sin. $F^{\circ} \times$ sec. $V^{e} \times$ sec. V^{f}
&c.	&c.

C

Because the lines Fig. 7. AE, EF, FG represent the weights of the several sections AB, BC, CD, the sum of those lines, or AG, will denote the sum of the weights of the sections $A + B + C$. And in general, if the angle of an abutment in an arch of equilibration should $= V^z$, and the angle of the first section $= A^\circ$, and its weight $= w$, the sum of the weights of the sections when adjusted, will $= w \times$ cotang. $A^\circ \times$ tang. V^z.

On this principle the weights of the sums of the successive sections, or the weights of the semiarches, will be as they are stated underneath.

Sums of the weights of the sections, or weights of the semiarches.

$A \; . \; . \; . \; . = w \times \text{cotang.}\, A^\circ \times \text{tang.}\, V^a = w \times \text{cotang.}\, A^\circ \times \text{tang.}\, A$

$A + B \; . \; . \; . = w \times \text{cotang.}\, A^\circ \times \text{tang.}\, V^b = w \times \text{cotang.}\, A^\circ \times \text{tang.}\, \overline{A + B}$

$A + B + C \; . = w \times \text{cotang.}\, A^\circ \times \text{tang.}\, V^c = w \times \text{cotang.}\, A^\circ \times \text{tang.}\, \overline{A + B + C}$

$A + B + C + D = w \times \text{cotang.}\, A^\circ \times \text{tang.}\, V^d = w \times \text{cotang.}\, A^\circ \times \text{tang.}\, \overline{A + B + C + D}$

&c. &c. &c.

The method of fluxions affords an additional confirmation of this proposition: suppose an arch adjusted to equilibrium to be composed of innumerable sections, the angles of which are evanescent; to ascertain the weight of the sum of these evanescent sections included within a given angle from the summit of the arch to the lowest abutment V^c; since the angles of the sections are evanescent, the quantity $V^c = V^d$: and for the same reason, the sin. of the angle D° will ultimately $= \dot{D}$. Wherefore, the evanescent weight of the section $D = r \times$ sin. $\dot{D} \times$ sec. $V^c = r \times \dot{D} \times$ sec. V^c. Let the tangent of the angle $V^c = x$ to radius 1; then the sec. of $V^c = \sqrt{1 + x^2}$; and because $V^c = V^d$, it follows that $V^c \times V^d = \overline{1 + x^2}$: the weight therefore of the evanescent section $D = w \times$ cotang. $A^\circ \times \dot{D} \times \overline{1 + x^2}$;

which is the fluxion of the weight of the arch equal to the fluxion of the angle D° sec.^{2} V^c × w × cotang. A°.

But the fluxion of an arc × into the square of its secant is known to be equal to the fluxion of the tangent of the same arc, when both quantities vanish together: therefore the integral or fluent, that is, the weight of the arch, will be equal to the tangent of the arc × into constant quantities; that is, the sum of the evanescent sections, or the weight of the entire arch, from the summit to the abutment = w × cotang. A × tang. V^c.

On the Model, No. 1, *for veryfying the Construction of an Arch, in which the Weights of the Sections* A, B, C, D, *&c. are inferred from the Angles given in the present Case* = 5° *each.*

Although the various properties of arches described in the preceding pages, respecting the weights and dimensions of the wedges, and their pressures against the abutments, require no further demonstration than what has been given in the preceding pages; yet, as it has been remarked, that philosophical truths, although demonstrable in theory, have often been found to fail when applied to practice; in order to remove every doubt of this sort, concerning the theory of arches, which is the subject of the preceding and present Dissertation, a model of an arch was constructed according to the conditions in Table I. in which the angle of each section = 5°, the weight of the first section = 1, the weights of all other sections being in proportion to unity. This arch, like most arches which were erected previously to the 16th century, consists of two semiarches, similar and equal and resting against each other, in the middle of the curve, as described in figure 2: the summit of the arch is occupied by two equal wedges A, A, resting against each other when coincident with the vertical plane VO; according to the construction of this proposition, the weight of the wedge A being assumed = 1, the weight of B appears to be 1.01542, and the weight of the wedge C = 1.04724. These weights being applied in the form of truncated wedges, supported upon immoveable abutments, sustain each other in exact equilibrium, although retained in their places by their weights and pressures only, and independently of any ties and fastenings which are usually applied in the case when the structure is intended for the purpose of sustaining superincumbent loads. The pressure between the two first sections in a direction parallel to the horizon $= p' = 11.24300$, the pressure against the lowest surface of the first section $= p = 11.47371$: the pressure on the lowest surface of the second section, or $B = q = 11.60638$: on the lowest surface of C, the pressure is $= r = 11.83327$. The intention of this model is not only to verify the properties of equilibrium of these wedges, acting on each other, but also to examine and prove the several pressures on the lowest surface of the sections to be in their due proportions, according to the theory here demonstrated. And it ought to be remembered that these pressures being perpendicular to the surfaces impressed, the reaction is precisely equal and contrary; for this reason, each of the surfaces subject to this pressure will have the effect of an abutment immoveably fixed.

The most satisfactory proof that the pressure on any abutment has been rightly assigned is, by removing the abutment and by applying the said force in a contrary direction; the equilibrium that is produced between forces acting under these circumstances, it is a sufficient proof that the reaction of the abutment is precisely equal to the force impressed upon it in a contrary direction.

After the weights of the several wedges in an arch of equilibration have been determined, in proportion to the weight of the first wedge A assumed to be $= 1$, some difficulty occurs in forming each wedge of proper dimensions, so that their weights shall be correspondent to the conditions required. A wedge being a solid body consisting of length, breadth, and thickness, of which one dimension, namely, the thickness, or depth, remains always the same; the weight of any wedge will be measured by the area or plane surface in each section, which is parallel to the arch; that is, if the thickness or depth of any section K (Fig. 7.) be put $= 1\frac{1}{2}$, the solid contents of the section K will be measured by the area K*tt*S multiplied into $1\frac{1}{2}$; put the angle SOT $= 5°$, the sin. of $2° 30' 0'' = s$, cos. $2° 30' 0'' = c$; also let $Ot = x$; then we find, by the principles of trigonometry, that the area $Ott = x^2 sc$, and the area OTS $= r^2 sc$, and the area $TttS = x^2 sc - r^2 sc$. Let the area corresponding to the weight of the section proposed $= k$, so that $x^2 - r^2 sc = k$; and $x^2 = \frac{k + r^2 sc}{sc}$: wherefore $x = \sqrt{\frac{k + r^2 sc}{sc}}$; and T*t* or S*t*, the slant height of the section K $= \sqrt{\frac{k + r^2 sc}{sc}} - r$. This being determined, the breadth of the section $tt = 2s \times ot = 2sx$, making therefore the radius OV $= 11.46281$, with the centre O, and the distance OV $= 11.46281$, describe the circular arc VABC; and in this arc from V set off the several chords VA, AB, BC, &c. $= 1$ inch, in consequence of which the angles VOA $=$ AOB $=$ BOC, &c. &c. will be $5°$ each. The slant height and the breadth of each section will be computed by the preceding rules.

On the Model, No. 2, *for illustrating and verifying the Principles of the Arch, when the Angle of each Section, after the first Section* A°, *are inferred according to the Rule in Page* 27 *of former Tract, from the Weights of the other Sections.*

In the propositions which have preceded, the several angles of the sections A°, B°, C°, D°, &c. have been considered as given quantities, from which the weights of the corresponding wedges have been inferred, both by geometrical construction and by calculation, when they form an arch of equilibration. The next inquiry is to investigate the magnitudes of the angles from having given the weights of the several sections; but as the construction and demonstration would not in the least differ from that which has already appeared in page 27 of the former Tract on Arches, it may be sufficient in the present instance to refer to the former Tract, both for explaining the principles of the construction and the demonstration, inserting in this place only the result, which is comprised in the following rule.

Having given A° the angle of the first section, and the weight $b = 1.25$ of the section B next following, together with the angle at which the lower surface of A is inclined to the vertical, called the angle of the abutment of the section A, or V^a, and the pressure on it $= p$, to ascertain the magnitude of the angle B°, in an arch adjusted to equilibrium: in the proposition referred to it is proved, that on the conditions stated, tang. $B° = \frac{b \times \cos. V^a}{p + b \times \sin. V^a}$ radius being $= 1$.

The model constructed to verify the principles of equilibration, consists of a circular arc drawn to a radius $= 21.7598$ inches. VA, AB, BC, &c. are chords $= 1$ inch each, and subtend at the centre of the circle angles of $2° \; 38' \; 0''$: as the angle of the first section $A^{\circ} = 2^{\circ}\,38'\,0''$, the angle of the abutment, or the angle contained between the vertical and the lowest surface of the section $A = V^{a} = 2^{\circ}\,38'\,0''$: the pressure on the lowest surface of $A^{\circ} = p = \frac{1}{\sin. A^{\circ}}$ $= 21.765553$, and according to the rule inserted in page 12, $\text{tang. } B^{\circ} = \frac{1.25 \times \cos. 2^{\circ}\,38'\,0''}{p + 1.25 \times \sin. 2^{\circ}\,38'\,0''} = 3^{\circ}\,16'\,29''$. Wherefore the angle of the abutment contained between the lowest surface of B and the vertical line $= A^{\circ} + B^{\circ} = 2^{\circ}\,38'\,0'' + 3^{\circ}\,16'\,29'' = 5^{\circ}\,54'\,29'' = V^{b}$. By the same rule, the angles of the successive sections C°, D°, E°, &c. &c. and the angles of the abutments corresponding, are computed as they are stated in the columns annexed, in page 17.

Let the arch to be constructed be supposed such as requires for its strength and security, that the weight or mass of matter contained in the lowest section R, shall be five times the weight of the first or highest section A, and let the arch consist of thirty-four sections, seventeen on each side of the vertical plane: on these conditions, the weight of the successive sections will be as follows: A = 1, B = 1.25, C = 1.50, D = 1.75, E = 2.00, F = 2.25, &c. as stated in Table IX: by assuming these weights for computing the several angles B°, C°, D°, &c. according to rule in page 12, they are found to be as in the ensuing columns, and the successive sums of the angles are the angles of the corresponding abutments. By considering the drawing of this model, it is found to contain the conditions necessary for calculating the areas required for estimating the weights of the voussoirs. For the inclination of each abutment to the abutment next following, is equal to the angle of the section which rests on the abutment; thus, the inclination of the lines I*i*, H*i*, is equal to the angle of the section I = H*i*I; also the inclination of the lines H*b*, G*b*, forms the angle of the section H = H*b*G, and so on.

Model No. 2.

Dimensions of an Arch of Equilibration: the Angle of the first Section, or A° = 2° 38′ 0″, and the Angles of the other Sections, and the Angles of the Abutments, are as follow:

Angles of the Sections.	Angles of the Abutments.
A° = 2° 38′ 0″	V^a = 2° 38′ 0″
B° = 3 16 29	V^b = 5 54 29
C° = 3 52 39	V^c = 9 47 8
D° = 4 24 36	V^d = 14 11 44
E° = 4 50 9	V^e = 19 1 53
F° = 5 7 16	V^f = 24 9 9
G° = 5 14 41	V^g = 29 23 50
H° = 5 12 14	V^h = 34 36 4
I° = 5 1 8	V^i = 39 37 12
K° = 4 43 23	V^k = 44 20 35
L° = 4 21 27	V^l = 48 42 2
M° = 3 57 33	V^m = 52 39 35
N° = 3 33 26	V^n = 56 13 1
O° = 3 10 21	V^o = 59 23 22
P° = 2 49 0	V^p = 62 12 22
Q° = 2 29 42	V^q = 64 42 4
R° = 2 12 31	V^r = 66 54 35

Geometrical Construction for drawing the Abutments, in the Model for illustrating Equilibrium of Arches, when the Magnitudes of the Angles are inferred from the Weights of the several Sections.

VABC, &c. represents the portion of a circular arc, which is drawn from the centre O, with the distance OV: VIO (Fig. 8.) is a line drawn perpendicular to the horizon, dividing the entire arch into two parts, similar and equal to each other: the radius OV = 21.7598 inches: from the point V, set off the chord VA = 1 inch, and the chords AB, BC, CD = 1 inch each; the angle of the first section will therefore be = 2° 38′ 0″: for as one half : 1 :: the sin. of $\frac{1}{2}$ A°, or sin. 1° 19′, to radius, which is, consequently, = 21.7598 inches: the semiarch VR consists of seventeen sections, the weights of which increase from 1 to 5, which is the weight of the lowest or last section; and from these conditions it is inferred, by the rule in page 15, that A° = 2° 38′ 0″, B° = 3° 16′ 29″, C° = 3° 52′ 39″, &c. the successive sums of these angles, or the angles of the abutments, A° = 2° 38′ 0″ = V^a, A° + B° = 5° 54′ 29″ = V^b, A + B + C = 9° 47′ 8″ = V^c, &c. as stated in Table IX.

The direction of the line must next be ascertained, determining the position of the abutment on which either of the sections, for instance the section I, is sustained: from the point O draw the line OI: it is first to be observed, that the angle contained between the line I and VO, or the angle VII = 39° 37′ 12″, according to the Table IX. and the angle VOI = 2° 38′ 0″ × by 9 = 23° 42′ 0″: make therefore the following proportion: as the sine of 39° 37′ 12″, is to the sine of VII — VOI = 15° 55′ 12″, so is radius OV, or 21.7598 inches to OI = 9.3597 inches; this being determined, if a line *iIt* is drawn through the point I, the line so drawn will coincide with the abutment on which the lowest surface of the section I is sustained; and by the same principle the directions of all the abutments are practically determined. Also it appears that the successive abutments I*i*, H*i*, include between them the angle H*i*I, which is therefore equal to the angle of the section I; therefore to find the solid contents measured by the area of the section I, the triangle *iss*, being made isosceles the area iss will be $= \frac{is^2 \times \text{sin. } sis}{2}$;* from which if the area IiH be subtracted, the remaining sum will be equal the area of the section I: put either of the lines $is = x$, then by the proposition which has been above mentioned, the area $iss = \frac{\overline{is}^2 \times \text{sin. } sis}{2}$, and by the same proposition, the area $HiI = \frac{iH \times iI \times \text{sin. } HiI^\circ}{2}$; consequently, is being put $= x$, we shall have $\frac{x^2 \times \text{sin. } I^\circ}{2} - Hi \times Ii \times \frac{\text{sin. } I^\circ}{2} = I$, it appears that $x^2 = \dagger \frac{2I + bI \times bH \times \text{sin. } I^\circ}{\text{sin. } I^\circ}$, and consequently $x = \sqrt{\frac{2I + \overline{bI \times bH} \times \text{sin. } I^\circ}{\text{sin. } I^\circ}}$: by the same rule the weights and dimensions of all the sections K, L, M, &c. are determined.

By the principles stated in the preceding pages, the weight of either of the highest sections in any course of voussoirs, together with the angle of the said section, regulates the magnitude of the horizontal thrust or shoot, and the perpendicular pressure on the ultimate or lowest abutment and the direct pressure against the lowest surface of any abutment will depend on the cotang. of the angle of the highest section and the sec. of the angle of the abutment jointly.

PROPOSITION.

* The area contained in a right-lined triangle ABC, Fig. 10, is equal to the rectangle under any two sides $\times \frac{1}{2}$ the sine of the included angle.

Let the triangle be ABC; AB and AC the given sides, including the angle BAC, between them.

Through either of the angles B draw BD perpendicular to the opposite base AC: by the elementary principles of geometry it appears, that the area of the triangle ABC = the rectangle under the base AC, and half the perpendicular height BD, or $\frac{AC \times BD}{2}$. But when BA is made radius, BD is the sine of the angle BAC: consequently, the line $BD = \frac{BA \times \text{sin. } A^\circ}{2}$; and the area $ABC = AC \times AB \times \frac{\text{sin. } CAB}{2} = \frac{AC \times AB \text{ sin. } A^\circ}{2}$, which is the proposition to be proved.

† I, here, means the weight of the section I.

In consequence of these properties, since each course of voussoirs stands alone, independent of all the voussoirs above and beneath, the strength of an arch will be much augmented by the degree of support afforded to the voussoirs situated in the course immediately above, as well as to those underneath, which may be connected with the former.

Moreover, the inconvenience is avoided which obviously belongs to the principles, that are sometimes adopted for explaining the nature of an arch, by which the whole pressure on the abutment is united in a horizontal line, contiguous to the impost; whereas the magnitude of the horizontal shoot, and the perpendicular pressure on the ultimate or lowest abutment has appeared by the preceding propositions to be proportioned to the weight of the highest section in the semiarch, and to the sec. of the angle of the abutment jointly; and consequently, the pressure on the different points of the abutment may be regulated according to any proportion that is required.

Whatever, therefore, be the form intended to be given to the structure supporting the road-way, and the weight superincumbent on an arch, no part of the edifice need to be encumbered by superfluous weight; on the contrary, such a structure, consisting of the main arch and the building erected on it, is consolidated by the principle of equilibrium, into one mass, in which every ounce of matter contributes to support itself, and the whole building.

The equilibration of arches being established by theory, and confirmed by experiment, it becomes a further object of experiment to ascertain, amongst the varieties of which the constructions of arches is capable, what mode of construction will be most advantageous, in respect to firmness and stability, when applied to any given case in practice. A simultaneous effort of pressure combined with weight, by which the wedges are pressed from the external towards the internal parts of an arch, being the true principle of equilibration, the wedges by their form endeavour to occupy a smaller space in proportion as they approach more nearly to the internal parts of the curve. It has appeared by the observations in page 29, Part I. that the bases of the sections may be of any lengths, in an arch of equilibration, provided their weights and angles of the wedges be in the proportions which the construction demands, observing only that if the lengths of the bases should be greatly increased, in respect to the depths, although, in geometrical strictness, the properties of the wedge would equally subsist, yet when applied to wedges formed of material substance, they would lose the powers and properties of that figure; this shews the necessity of preserving some proportion between the lengths of the bases and depths of the wedges, to be determined by practical experience rather than by geometrical deduction.

With this view, a further reference to experiment would be of use, to ascertain the heights of the sections or voussoirs, when the lengths of the bases are given, also when the angles B°, C°, D°, &c. are inferred from the weights of the sections considered as given quantities, to ascertain the alterations in the angles B°, C°, D°, &c. from the summit of the arch, which would be the consequence of varying the angle of the first section A°, so as to preserve the equilibrium of the arch unaltered: by referring to Table VI. we observe, that when the weights of the sections are equal to each other, or A = B = C = D, &c. and the angle of the first section $= 5°$; then to form an arch of equilibration, the angle of the second section, or B° must $= 4° \, 55' \, 30''$, the angle of the third section C° $= 4° \, 46' \, 53''$, &c. And it becomes an object of experimental examination how far the stability and firmness of an arch will be affected by any alterations of this kind, and to judge whether in disposing a given weight or mass of matter (iron for instance) in the form of an arch, any advantage would be the consequence of constructing the sections so that the first section will subtend an angle of 1°, 2°, 3°, 5°, or any other angle at the centre of the arch, all other circumstances being taken into account. When the angle of the first section = 5°, and the weights of the successive sections = 1, the angles of the abutments will be severally $V^a = 5° 0' 0''$, $V^b = 9° 55' 30''$, $V^c = 14° 42' 23''$, and so on, as stated in Table VI. By referring likewise to Table VIII. we find the angle of the first section assumed = 1°, and the weight of each of the subsequent sections being = 1, the angles of B°, C°, &c. are severally $B° = 1° 4' 57''$, $C° = 1° 9' 51''$, $D° = 1° 14' 39''$; consequently, the angles of the abutments will be as follows: $V^a = 1°$, $V^b = 2° 4' 57''$, $V^c = 3° 14' 48''$, $V^d = 4° 29' 28''$, &c. which give the dimensions of the sections when they form an arch of equilibration.

It has been frequently observed, by writers on the subject of arches, that a thin and flexible chain, when it hangs freely and at rest, disposes itself in a form which coincides, when inverted, with the form of the strongest arch. But this proposition is without proof, and seems to rest on some fancied analogies arising from the properties of the catenary curve, rather than on the laws of geometry and statics, which are the bases of the deductions in the two Dissertations on Arches, contained in the preceding pages; if it should be proved that an arch built in the form of a catenary or other specific curve, acquires, in consequence of this form, a superior degree of strength and stability, such proof would supercede the application of the properties demonstrated in these Dissertations.

Concerning the relative Positions of the Centres of the Abutments and the Centre of the Circle.*

When the angle of an abutment is greater than the corresponding angle at the centre of the circle; in this case, the centre of the abutment falls above the centre of the circle, as in Fig. 9. When the angle of the abutment is less than the angle at the centre of the circle, the centre of the abutment falls beneath the centre of the circle, as represented in Fig. 9. When the angle of the abutment is equal to the angle at the centre, this case will coincide with that which is stated in pages 4 and 5 preceding, in which $V^a = A°$, $V^b = 2\,A°$, $V^c = 3\,A°$, &c. $V^b = A°$, $V^l = 2\,A°$, $V^b = A°$, &c. &c. and consequently the centre of the abutment coincides with the centre of the circle.†

Further Observations on the Courses of Voussoirs.

A, B, C, D, E, &c. terminating the letter F, denote the sections which form the first course of voussoirs in a semiarch of equilibration, of which A° is the first, or one of the highest, sections: if the weight of the section A be $= w$, and the angle of the abutment VOF $= V^c$. then it has appeared, by the preceding pages, that the pressure against the lowest or ultimate abutment $= w \times$ cotang. A $\times$ sec. V^c. 2dly. Let B° be the angle of the first section in the next course of voussoirs, terminated on each end by the letter L, and let y be the weight of the first section, the

* The point in which any abutment intersects the vertical line is called, in these pages, the centre of that abutment.

† Let VO be a line drawn through V, the middle point of the arch passing through the centre of the circle O; on this arc the angles of the sections and the angles of the abutments are measured: p, the point where any abutment, for instance I, continued intersects the vertical vO, is called the centre of the abutment.

pressure on the last or ultimate abutment $= x \times$ cotang. B $\times$ sec. V^c. Moreover, let z be the weight of the first section C, in the third course of voussoirs, which is terminated by the letter P. It follows that the proportions of pressure on the ultimate abutment denoted by the letters F, L, P, will be $w \times$ cotang. $A^o \times$ sec. $V^c + x \times$ cotang. $B^o \times$ sec. V^c, and $y \times$ cotang. $C^o \times$ sec. V^c respectively, and according to these quantities, the respective pressures on the several parts of the abutment, will be regulated according to any law that may be required.*

The principles of arches having been established according to the preceding theory, and confirmed by experiment, described in the experiments No. 1 and 2; in the first of these, the angles of each section are constructed $= 5°$, and the weight of the section A having been assumed $= 1$, the weights of the sections B°, C°, D°, &c. are inferred as stated in Table I. from the angles B°, C°, D°, &c. considered as given quantities. In No. 2, the angle of the first section is assumed $= 2° \, 38' \, 0''$. The remaining angles are inferred from the given weights by the rule in page 15, $A = 1.00$, $B = 1.25$, $C = 1.50$, &c. to $Z = 5$, which is the weight of the lowest or ultimate section. It has appeared in page 29, in the former Tract, that whatever be the figure of the interior curve corresponding in an arch of equilibration, the bases of the sections which are disposed in this form may be of any lengths, provided the weights and the angles of the sections are in the proportions which the construction demands.

CORRECTION OF THE ENGRAVING FIG. 6.

* That the engraving of the Figure 5 may correspond with the text, the summit of the first course of voussoirs ought to be marked A, the first section of the second course should be marked B, and of the third the first section = C, and so on; this will make the text correspondent with the Figure 6.

A further reference to experiment would be of use in practical cases, to ascertain how far the strength and stability of an arch would be affected by altering the proportion between the lengths of the voussoirs and the heights thereof; for instance, when the lengths of the wedges are given to ascertain the alterations in the stability of the arch when the depths or heights of the sections are three, four, or five times the length. Let the following case be also proposed; the entire weight of an arch being supposed known, what part of this entire weight must the first section consist of, so as to impart the greatest degree of strength to the structure; also to decide whether the angle of the first section ought to be made 1°, 5°, 10°, &c. or of what ever magnitude would contribute to the same end. To these may be added the following cases to be discussed; when the angles of the several sections are inferred from the weights thereof, to investigate what must be the proportion of the said weights, so as to make the arch uniformly strong throughout.

FURTHER CONSIDERATIONS CONCERNING THE CONSTRUCTION OF THE MODELS No. 1 AND No. 2.

Dimensions of a Model No. 1, *of an Arch of Equilibration. Radius* = OV = 11.46281, *the Angle of each Section* = 5°, *the Chord of each Arch* = 5° = 1 *Inch.* (Fig. 7.)

The first section is a brass solid, the base of which = KV = 1 inch, and the sides V*v*, K*k*, or the slant height of the section A = .961, and the depth or thickness of each section = $1\frac{1}{2}$ inch, the breadth of A or *vk* = 1.084.

The weights of the sections, as they are calculated according to Table No. I, the first section being assumed as unity.

E

Rule for making the brass voussoirs equal to the weights which are expressed in Table No. I. Let the sine = $2^\circ\ 30'\ 0'' = s$, the cosine c when radius = 1; then making the radius $= r$, the area of the triangle $vOk = r^2 \times sc$, and the area $VOK = \overline{VO}^2\, sc$; from whence Vv, or the slant height of the section A, when the weight = 1, is found to be $\sqrt{\frac{1 + r^2 sc}{sc}} - OV = .961$, the breadth $vk = 2s \times Ov = 1.084 = Vk$. Thus, by the same rule, the slant height of the section $B = \sqrt{\frac{B + r^2 sc}{sc}} - r = .9749$, and the breadth $ll = 1.084$, in all the sections entered in Table I. are calculated.

MODEL, No. I.

Sections.	Weights of the sections as they are calculated in Table No. I. in the Tract on Arches.	Slant height of the sections.	Breadth of the sections.	Weights of the sections when made of brass, specifically heavier than water, in proportion 8 to 1: in ounces avoirdupois.	Weights of the sections when made of brass, in lbs. avoirdupois.	Sums of the weights in lbs. avoirdupois.
A	= 1.00000	K*k* = .961	*vk* = 1.084	A = 6.9444	0.43403	0.43403
B	= 1.01542	L*l* = .974	*ll* = 1.085	B = 7.0515	0.44072	0.87475
C	= 1.04724	M*m* = 1.004	*mm* = 1.087	C = 7.2725	0.45453	1.32928
D	= 1.09752	N*n* = 1.050	*nn* = 1.092	D = 7.6215	0.47535	1.80564
E	= 1.16972	O*o* = 1.116	*oo* = 1.097	E = 8.1230	0.50769	2.31333
F	= 1.26922	P*p* = 1.207	*pp* = 1.105	F = 8.8140	0.55087	2.86421
G	= 1.40427	Q*q* = 1.329	*qq* = 1.116	G = 9.7518	0.60949	3.47370
H	= 1.58754	R*r* = 1.492	*rr* = 1.130	H = 11.024	0.69903	4.16274
I	= 1.83910	S*s* = 1.713	*ss* = 1.149	I = 12.771	0.79822	4.96096
K	= 2.19175	T*t* = 2.016	*tt* = 1.176	K = 15.220	0.95128	5.91224
L	= 2.70196	V*v* = 2.444	*vv* = 1.213	L = 18.764	1.17419	7.08643
M	= 3.47366	U*u* = 3.067	*uu* = 1.264	M = 24.122	1.50619	8.59263
N	= 4.71440	W*w* = 4.098	*ww* = 1.357	N = 32.738	2.04618	10.63882
O	= 6.89199	X*x* = 5.553	*xx* = 1.484	O = 47.860	2.99130	13.63012
P	= 11.25371	Y*y* = 8.279	*yy* = 1.722	P = 78.150	4.88443	18.51455
Q	= 22.16552	Z*z* = 13.836	*zz* = 2.207	Q = 153.92	9.62045	28.13500
R	= 65.8171	A*a* = 29.056	*aa* = 3.534	R = 456.96	28.57000	56.70500

On the Construction of the Model No. 2, *in an Arch of Equilibration, in which the Angles of the several Sections are inferred from the Weights thereof, according to the Rule in Page* 15.

In this model the arc A, B, C, &c. is a portion of an arc of a circle: the first section A subtends an angle at the centre of the circle $A^\circ = 2^\circ\ 38'\ 0''$, the chord of which = 1 inch = to the chord of BC, CD, DE, &c. radius = OV = 21.7598 inches: the weight of the first section being assumed = 1, the weights of the sections B, C, D, &c. are considered as proportional to the weight of the first section when it is = 1; if the weight of the seventeenth section or R = 5, the weights of the intermediate sections will be B = 1.25, C = 1.50, D = 1.75, &c. as stated in Table IX: and since A° the angle of the first section $= 2^\circ\ 38'\ 0''$, by applying the rule demonstrated in page 27 in former Dissertation, and referred to in page 15 of this Tract, the angles of the several sections are found to be $A^\circ = 2^\circ\ 38'\ 0''$, $B^\circ = 3^\circ\ 16'\ 29''$, $C^\circ = 3^\circ\ 52'\ 39''$, and the corresponding angles of the abutments, or successive sums of the angles of the sections, are $2^\circ\ 38'\ 0'' + 3^\circ\ 16'\ 29'' = 5^\circ\ 54'\ 29'' = V^b$. Moreover, $A^\circ + B^\circ + C^\circ = 9^\circ\ 47'\ 8'' = V^c$, and thenceforward according to the same law of progression. The next object of inquiry is, to ascertain from what point I in the line OV the line OII must be drawn, so as to coincide with the lowest surface of the section I, when inclined to the vertical at the given angle VII. The angle subtended by the semiarch VI at the centre O is measured by the angle IOI, and the difference of these angles, or VII — IOI = IIO. The radius IO being denoted by the same letters which distinguish the line IO, the different meaning will be determined by the context. From the principles of trigonometry, the following proportion is inferred; as IO : OI :: the sin. of IIO

E 2

to the sin. of OII or VII; consequently, the line $IO = \frac{OV \times \sin. OII}{VII}$. As an example, let it be required to ascertain the inclination of the abutment to the vertical, on which the section I is sustained when it forms a portion of an arch of equilibration, and the angle of the abutment VII = 39° 37′ 12″: the angle VIO subtended by the semiarch VI at the centre of the circle = 23° 42′ 0″, which being subtracted from the angle of the abutment 39° 37′ 12″, leaves the angle IOI = 15° 55′ 12, and the distance required from the centre, $OI = OV \times \frac{\sin. OII}{\sin. VII}$, or because OV = 21.7598 inches, OI = 9.35978 inches; making, therefore, the line OI = 9.35978 inches, through the points II draw the line I, I, *t*, which will be the position of the abutment on which the section I rests, the angle of which, V^i = VII, is the inclination of the abutment V^i to the vertical: for the same reason VHH = the angle of the abutment V^b = VHH, the difference of these two angles VII − VHH = G*b*H, or the angle of the section H°: making, therefore, the line G*b* = *a*, H*b* = *b*, the properties of trigonometry give the area of the triangle G*b*H $= ab \times \frac{\sin. H}{2}$; on the same principle, the area of the triangle $HiI = HiI = Hi \times Ii \times \frac{\sin. H^o}{2}$; and thus the areas of all the triangles will be measured, from having given the sides of the triangles and the angles included between them. The sides of the triangles may be measured by a scale of equal parts, as stated in Table I. and in this manner the sides of all the triangles were correctly measured by Mr. Berge, so as not to err from the truth by more than an unit in the fourth decimal place. This measurement was essential for computing the distance of the vertex from the base, so as to form the dimensions of the brass wedges, correctly and independently of their weights, in each triangle. For instance, *r*Q being put = *a* and *r*R = *b*, this will give the area of the triangle

$rQR = ab \times \frac{\text{sin.}R^\circ}{2}$; and if the triangle raa is made isosceles, or $ra = x$, the area of the triangle $Raa = \frac{x^2 \times \text{sin.}R^\circ}{2}$ — the area $\frac{b \times a \times \text{sin.}R^\circ}{\text{sin.}R^\circ}$; or if the difference of the areas is put $= w$, the result will be $\frac{x^2 \times \text{sin.}R^\circ}{2}$ — the area $\frac{b \times a \times \text{sin.}R^\circ}{2} = w$, or $x = ra = \sqrt{\frac{2w + \text{the area } a \times b \times \text{sin.}R^\circ}{\text{sin.}R^\circ}}$; wherefore $Qa = \sqrt{\frac{2w + a \times b \times \text{sin.}R^\circ}{\text{sin.}R^\circ}} = 28.5777 - rQ$ and $Ra = \sqrt{\frac{2w + a \times b \times \text{sin.}R^\circ}{\text{sin.}R^\circ}} - rR$.

Thus, by actual measurement, $a = 23.9248$ inches, and $b = 24.3056$, and the area $ab \times \frac{\text{sin.}R^\circ}{2} = 10.73670 = \sqrt{\frac{2w + a \times b \times \text{sin.}R^\circ}{\text{sin.}R^\circ}} - Qr$, and $Ra = \sqrt{\frac{2w + a \times b \times \text{sin.}R^\circ}{\text{sin.}R^\circ}} - rR$: the area $raa = \frac{x^2 \times \text{sin.}R^\circ}{2} = 15.73670$, or the area $raa = 15.73670$; the result is, that the area $aaRQ = raa - rQR = 5$ square inches: and since every square inch of area is occupied by a weight of a section $= 6.9444$ oz. avoirdupois, we arrive at the following conclusion, that the weight of the section $R = 5 \times 6.9444 = 34.7222$ oz. avoirdupois. Because $\sqrt{\frac{2w + a \times b \times \text{sin.}R^\circ}{\text{sin.}R^\circ}} = ra = 28.57770$, this determines both the greater and lesser sides of the section R; namely, the greater side being $= ra - rQ = 4.6529$; and the lesser side being $= ra - rR = 4.2721$ inches; in this way, the Table is formed, shewing the greater and lesser sides of the several sections.

According to this mode, the dimensions of all the brass wedges were formed; the investigation of the angles of the wedges from the weight thereof is the subject of investigation in page 27 of the First Part of the Tract, entitled a Dissertation on the Construction and Properties of Arches; and it appears that if the angle of the first section is given $= A^\circ$, together with the weight thereof $= a$, assumed to be = 1, the weights of the other section B = b = 1.25, the weight of C = c = 1.50, of D = d = 1.75, &c. The principle of equilibrium is established, by making the tang. of the angle $B^\circ = \frac{a \times \cos. A^\circ}{p + a \times \sin. A^\circ}$, also the tang. of the angle $C^\circ = \frac{b \times \cos. B^\circ}{q + b \times \sin. B^\circ}$, as they are stated in Table IX. which contains the conditions, founded on supposing that the strength and security of the arch are such as require that whatever weight should be contained in the first section, the weight of the seventeenth section R shall be five times as great: making, therefore, the weight of A = 1, the weight of B = 1.25, and C = 1.50, and the weight of the seventeenth section or R = 5, &c. Thus the angle of the first section A° being assumed = 2° 38′ 0″, and the initial pressure on the lowest surface of A = p = 21.76555, and the weight of the first section = a = 1: from these data the following results are obtained: $\frac{a \times \cos. V^\circ}{p + a \times \sin. V^\circ}$ = 2° 38′ 0″ tang. $B^\circ = \frac{b \times \cos. V^a}{p + b \times \sin. V^a}$ = 3° 16′ 29″ tang. $C^\circ = \frac{c \times \cos. V^b}{q + c \times \sin. V^b}$ = 3° 52′ 39″, &c. &c. according to the statement in Table IX.

The Dimensions of the Sections, according to the Rule in Page 29.

	Lesser Sides.	Greater Sides.		Lesser Sides.	Greater Sides.
A	= 0.97827	0.97827	K	= 2.73754	3.12824
B	= 1.20676	1.21126	L	= 3.14849	3.47069
C	= 1.41568	1.44928	M	= 3.36800	3.71700
D	= 1.61833	1.66693	N	= 3.64620	4.01460
E	= 1.81718	1.90118	O	= 3.87463	4.25476
F	= 2.00676	2.13656	P	= 4.16762	4.55142
G	= 2.20080	2.36920	Q	= 4.43768	4.82038
H	= 2.42850	2.63910	R	= 4.27210	4.65290
I	= 2.62584	2.88134			

Model, No. II.

Table I. Lines measured by Mr. Berge, on a brass plate, being the distances as undermentioned.				Sections	II. Breadth of the sections.		III. Areas of the greater triangles.		IV. Areas of the lesser triangles.	
OV	= 21.7598	OA	= 21.7598	A	*vk*	= 1.0449	O*vk*	= 11.8770	OVA	= 10.8770
*b*A	= 17.4925	*b*B	= 17.4970	B	*ll*	= 1.0689	*bll*	= 9.9918	*b*AB	= 8.7418
*c*B	= 14.8136	*c*C	= 14.8372	C	*mm*	= 1.1010	*cmm*	= 8.9315	*c*BC	= 7.4315
*d*C	= 13.0092	*d*D	= 13.0578	D	*nn*	= 1.0997	*dnn*	= 8.2809	*d*CD	= 6.5309
*e*D	= 11.7886	*e*E	= 11.8726	E	*oo*	= 1.1550	*eoo*	= 7.8994	*e*DE	= 5.8994
*f*E	= 11.0657	*f*F	= 11.1955	F	*pp*	= 1.1796	*fpp*	= 7.7791	*f*EF	= 5.5291
*g*F	= 10.7424	*g*G	= 10.9108	G	*qq*	= 1.2007	*gqq*	= 7.8569	*g*FG	= 5.3569
*h*G	= 10.5917	*h*H	= 10.8023	H	*rr*	= 1.2012	*hrr*	= 7.9387	*h*GH	= 5.1887
*i*H	= 10.9460	*i*I	= 11.2015	I	*ss*	= 1.2118	*iss*	= 8.3632	*i*HI	= 5.3632
*k*I	= 11.6478	*k*K	= 11.9385	K	*tt*	= 1.1898	*ktt*	= 8.9881	*k*IK	= 5.7381
*l*K	= 12.0988	*l*L	= 12.4210	L	*vv*	= 1.1838	*lvv*	= 9.2090	*l*KL	= 5.7090
*m*L	= 13.3775	*m*M	= 13.7265	M	*uu*	= 1.1814	*muu*	= 10.0883	*m*LM	= 6.3383
*n*M	= 14.7270	*n*N	= 15.0954	N	*ww*	= 1.1961	*nww*	= 10.8966	*n*MN	= 6.8966
*o*N	= 16.6668	*o*O	= 17.0472	O	*xx*	= 1.1584	*oxx*	= 12.1119	*o*NO	= 7.8619
*p*O	= 18.6297	*p*P	= 19.0135	P	*yy*	= 1.1135	*pyy*	= 13.2031	*p*OP	= 8.7031
*q*P	= 21.0620	*q*Q	= 21.4447	Q	*zz*	= 1.1269	*qzz*	= 14.5810	*q*PQ	= 9.8310
*r*Q	= 23.9248	*r*R	= 24.3056	R	*aa*	= 1.1014	*raa*	= 15.7367	*r*QR	= 10.7367

MODEL, No. II.

V. Differences between the greater and lesser triangles.	Sections.	Given weights of the sections.	VI. Pressures on the lowest surface of each section.	VII. Angles of the sections.	VIII. Angles of the abutments.	IX. Distances of the vertex from the base of each triangle Ovk.
1.0000	A = a	= 1.00	p = 21.76555	A° = 2° 38′ 0″	V^a = 2° 38′ 0″	Ok = 22.73807
1.2500	B = b	= 1.25	q = 21.85867	B° = 3 16 29	V^b = 5 54 29	bl = 18.70376
1.5000	C = c	= 1.50	r = 22.06356	C° = 3 52 39	V^c = 9 47 8	cm = 16.25288
1.7500	D = d	= 1.75	s = 22.42739	D° = 4 24 36	V^d = 14 11 44	dn = 14.67613
2.0000	E = e	= 2.00	t = 22.99972	E° = 4 50 9	V^e = 19 1 53	eo = 13.68978
2.2500	F = f	= 2.25	v = 23.82853	F° = 5 7 16	V^f = 24 9 9	fp = 13.20226
2.5000	G = g	= 2.50	u = 24.95590	G° = 5 14 41	V^g = 29 23 50	gq = 13.11160
2.7500	H = h	= 2.75	w = 26.41465	H° = 5 12 14	V^h = 34 36 4	hr = 13.23080
3.0000	I = i	= 3.00	x = 28.22645	I° = 5 1 8	V^i = 39 37 12	is = 13.82734
3.2500	K = k	= 3.25	y = 30.40220	K° = 4 43 23	V^k = 44 20 35	kt = 14.77604
3.5000	L = l	= 3.50	z = 32.94376	L° = 4 21 27	V^l = 48 42 2	lv = 15.56949
3.7500	M = m	= 3.75	a = 35.84656	M° = 3 57 33	V^m = 52 39 35	mu = 17.09450
4.0000	N = n	= 4.00	b = 39.10209	N° = 3 33 26	V^n = 56 13 1	nw = 18.74160
4.2500	O = o	= 4.25	c = 42.69992	O° = 3 10 21	V^o = 59 23 22	ox = 20.92166
4.5000	P = p	= 4.50	d = 46.62917	P° = 2 49 0	V^p = 62 12 22	py = 23.18112
4.7500	Q = q	= 4.75	e = 50.87939	Q° = 2 29 42	V^q = 64 42 4	qz = 25.88238
5.0000	R = r	= 5.00	f = 55.44104	R° = 2 12 31	V^r = 66 54 35	ra = 28.57770

From the preceding observations, the following practical rules may be inferred for deducing, in general, the weights of the sections, the pressures on the lowest surfaces thereof, and the weights of the semiarches, from the conditions on which they depend: to give a few examples of each rule, are applied to the Tables subjoined to this Treatise: it appears from page 10, that the weight of any section is equal the product formed by multiplying the weight of the first section, (assumed $= w$) into the cotang. of the first section, × into the sine of the angle of the given section × secant of the angle of the abutment of the preceding section, × secant of the angle of the abutment of the section given: in this manner the weight of the section R in Table No. I. may be found: for w being $= 1$, and the angle of the first section $= 5°$, the cotang. of $5° = 11.430052$, and the angle of the section R $= 5°$, sin. $5° = .0871557$: the angle of the abutment of the section preceding $= V^q = 80°$, and the angle of the abutment of the section given $V^r = 85°$: the result is, that the weight of the section R $= 11.430052 \times .0871557 \times 5.7587705 \times 11.473713 = 65.8171$. By page 10 it also appears, that the pressure upon the lowest surface of any section R is equal to the product which arises from multiplying the weight of the first section × cotang. of the angle of the first section × by the secant of the angle of the abutment of the given section, which makes the pressure on the lowest surface of the section R $= 11.430052 \times 11.473713 = 131.1450$, agreeing with the number entered opposite to the section in the column entitled entire pressures.

Lastly, the sum of the weight of the sections is found to be cotang. $A° = 11.430052 \times$ tang. $85° = 130.6401$, when the weight of the first section is $= 1$, agreeing with the number entered in Table No. I. opposite Sr. By similar rules applied to the several

F

Tables II, III, IV, V, &c. the results will be found to correspond with those entered in the respective Tables.

In the Table No. IV. the angles of the sections are taken indiscriminately and at hazard; but the rules which have been exemplified above, in the former cases, will be no less applicable to the computation of the numbers in all the Tables. In the Table No. IV. the angle of the section O = 12°, the weight of the section O = 281.4682; to compare this with the rule; the weight ought to be = w × cotang. 5 × sin. 12° × sec. 76 × sec. 88° = 281.4682, as above stated: also by the rule in page 10, the pressure on the lowest surface of O = w × cotang. 5° × sec. 88° = 327.5108, corresponding with the pressure, as stated in Table IV. Also in this Table the angle of the section P = 1°, and the angle of the abutment V^p = 89°, the angle of the abutment of the section O or V^o = 88°, the other notation remaining as before, the weight of the section P = 327.5107, and the pressure on the lowest surface of P = 654.9206, the weight of the semiarch = w × cotang. 5° × tang. 89 = 654.8220, as entered in Table IV. The computations founded on these rules produce results in no case less correct than in the former instances.

In No. VIII. the angle of the first section = 1°, and the angle of the section R^o = 1° 54′ 18″.421; the angle of the abutment of the same section (R) = 26° 18′ 54″.747: from these data, the rule above mentioned gives the weight of the section R = w × cotang. 1° × sin. 1° 54′ 18″.421 × sec. 24° 24′ 36″.316 × sec. 26° 18′ 54″.747 = 2.33333, which is the correct weight of the section R, as entered in Table VIII. To find the weight of the section R in Table IX. according to this rule, the weight of the section R = cotang. 2° 38′ 0″ × sin. 2° 12′ 31″ × sec. 64° 42′ 4″ × sec. 66° 54′ 35″ = 5.00000, as entered in Table IX.

It is needless to multiply examples to the computation of these Tables, the numbers in all cases being equally correct with those in the preceding instances, by which the rules for computing the Tables have been abundantly verified.

Experiment for determining the horizontal Pressure in Model No. 1.

In considering the circular arch as completed, it is difficult, at first view, to ascertain the magnitude of pressure sustained by any of the surfaces on which the sections are supported. Both the theorists and practical architects have differed considerably concerning this point. From the preceding demonstrations, and the ensuing experiment, it appears, that the magnitude of pressure sustained by the vertical plane is to the weight of the first section as the cotang. of 5° is to radius; and the weight of the first section, or w, having been found $= .43403$ parts of an avoirdupois lb. and the cotang. of 5° being $= 11.430052$; the result is, that the horizontal force or pressure $= .43403 \times 11.430052 = 4.961$ lbs. avoirdupois, differing very little from 5 lbs. which, in this experiment, counterbalances the horizontal pressure.

A second Experiment on the Model No. 1.

If the brass collar is placed round the section C, so that the line *cd* may pass over the fixed pulley in the direction *cd*, the equilibrium weight in this case being $= w \times$ cotang. 5° sec. 15°, or $.43403 \times 11.430052 = 5.1360$ lbs. avoirdupois, being suspended at the extremity of the line, keeps the whole in equilibrio.

Horizontal Force, by Experiment on Model No. 2.

In this experiment all the sections on one side of the vertical line or plane being taken away, and a force $= 11$ lbs. weight is suspended at the extremity of the line *c d* passing over the pulley

x, in a direction parallel to the horizon; after the Model and centre arch have been adjusted, as in the last experiment, when the centre arch is taken away, the remaining sections will be sustained in equilibrio.

A second Experiment on the Model No. 2.

The brass collar being placed round the section C, and a weight of $12\frac{1}{2}$ lbs. is applied to act on the lowest surface of the section C, when the brass central arch is removed, all the sections in the remaining half of the arch wil be sustained, without further dependance on the brass central arch.

On the Experiments for illustrating the Propositions concerning the Pressures on the lowest Surface of each Section, and against the vertical Surface, in an Arch of Equilibration.

In the Model No. 1, the angle of the first section $A^\circ = 5^\circ$, and it appears from the preceding propositions, that in this case, the horizontal force or shoot, as it is called, $= w \times$ cotang. 5°, in which expression w is equal the weight of $1\frac{1}{2}$ cubic inches of brass, the specific gravity of brass is to that of water in the proportion of about 8 to 1, and the weight of a cubic inch of water is very nearly $= .57870$ ounces avoirdupois;* it will follow, that the weight of a cubic inch and half of brass will be $.57870 \times 1\frac{1}{2} \times 8 = 6.9444$ ounces, or 0.43402 parts of a pound avoirdupois.—If all the sections on one side of the arch are removed, and a force in a horizontal direction is applied, that is in a direction perpendicular to the vertical surface of the first section, the whole will be kept

* By a decisive experiment of Mr. Cotes it appeared, that the weight of a cubic foot of pure rain water was exactly 1000 ounces avoirdupois; therefore, since the magnitude of a cubic foot $=$ 1728 cubic inches, the weight of a cubic inch of rain water $= \frac{1000}{1728}$ $= .57870$ ounces avoirdupois.—Cotes's Hydrostatics, p. 43.

in equilibrio by a force of 5 pounds avoirdupois, consisting of the equilibrium weight, which is 4.961 added to a friction weight, amounting to 0.039, being a weight exactly sufficient to counteract the effects of friction, cohesion, and tenacity.

Experiment for determining the horizontal Force or Pressure in the Model No. 2, *in which the Weight of the first Section* = .43403 *Parts of an avoirdupois lb. and the Angle of the first Section* = 2° 38′.

If half the number of sections on one side of the arch in Model No. 2. are removed, and a force of 11 pounds weight, acting in a direction parallel to the horizon, is applied to sustain the other half of the arch, the whole will be kept in equilibrio by a weight of 9.437 added to a weight of 1.563, making altogether the weight of 11 pounds avoirdupois.

On the general Proportion of the Pressures on the lowest Surface of each Section in the Model No. 1, *expressed in general by* $w \times$ *Cotang*. A° × *Sec*. V^c.

In the case of the pressure on the section C = w × cotang. A° × sec. V^c: here w = 0.43402 pounds; the angle of the abutment = 15°, the secant of which = 1.0352762, and the cotang. of 5° being = 11.430052, the pressure on the lowest surface of the section C = 5.1359, the equilibrium weight, when all the sections below the section C are removed, in the Model No. 1, and the weight of $5\frac{1}{2}$ pounds is applied against the lower surface of C, the friction weight being = 0.3641, when the brass central arch is removed, the whole will be sustained in equilibrio.

Similar Experiment upon the Model No. 2

The weight of w, that is, the weight of the first section in Model No. 2, is the same with the weight of w in Model No 1;

that is, $w = 6.9444$ ounces, $= 0.43402$ pounds avoirdupois; which is the weight of $1\frac{1}{2}$ cubic inch of brass; and, by the rule in page 10, the pressure on the lowest surface of $C = w \times$ cotang. $2° 38' 0'' \times$ sec. $V^c = 9.5762$.* If, therefore, all the sections below C are removed, and a weight of $12\frac{1}{2}$ pounds is applied against the lowest surface of C, when the centre brass arch is taken away, the remaining arch will be sustained in equilibrio.

By a similar experiment, the proper weight $= w \times$ cotang. $A^o \times$ sec. V^z applied in a direction against the lower surface of any other section Z, or perpendicular to it, would have the effect of sustaining it in equilibrio.

It has been remarked, in the First Part of this Tract, (page 5.) that if the materials of which an arch is constructed were perfectly hard and rigid, so as not to be liable to any change in their form, and the abutments were removably fixed; an arch, when the sections have been adjusted to equilibration, although little deviating from a right line, would be equally secure, in respect to equilibrium, with a semicircular or any other arch. This observation applies in some degree to the construction of a rectilinear or flat arch, according to a method employed by engineers, for transmitting water through the cavities of the several sections, each of which, when filled with water, will be nearly of the same weight; and for this reason it would be expedient to adopt the plan of construction which is numerically represented in Table VI. or one of the various other plans, in each of which the weights of each section are assumed $= 1$.

Construction of a Rectilinear Arch. Fig. 11.

COC represents a horizontal line, in which the lines OA, AB, BC, &c. are set off at equal distances from each other. From the

* $w = .434027$ cotang. $2° 38' 0'' = 21.742569$ sec. $V^c = 1.014763$ $w \times$ cotang. $A^o \times 9° 47' 8'' = 9.5762$.

point O, considered as a centre, draw O*a* inclined to the line OV, at an angle of 5°: through the point O likewise draw O*b*, inclined to OV, at the angle 9° 55′ 30″; also through the point O draw O*c* inclined to OV, at an angle = 14° 42′ 23″; and draw through the point A*a* parallel to O*a*, through B draw B*b* parallel to O*b*; likewise through C draw C*c* parallel to O*c*, &c. these lines, representing thin metallic plates, of which the angles are 5°, 4°, 55′ 30″, 4° 46′ 53, &c. respectively; and the sections OV, A*a*, B*b*, C*c*, &c. being formed of dimensions similar and equal to the sections on the other side; that is, VO, *aa*, forming an angle of 5°; A*a*, B*b*, 4° 55′ 30″; and B*b*, C*c*, an angle of 4° 46′ 53″, &c. the whole will constitute a rectilinear arch of equilibration, supporting itself in equilibrio by the help of small assistance from beneath, and admitting the water to pass freely through the cavities of the sections.

The geometrical figures were drawn to a scale equal to the original Model; that is, the radius of Fig. 7. was 11,46281 inches, and the radius of the Model No. 2. = 21.7598 inches; the engraving of these drawings are in proportion to those numbers; that is, Fig. 7. and in the Fig. 8. in the proportion of 1 to 3. It may be added, that the Figure 9. was drawn to a radius = 10 inches, which is engraved in proportion of $\frac{1}{2}$, or to a radius = 5 inches.

The radius = OV (Fig. 8.) in the original drawing is = 21.7598 inches, and OQ is, by Table X. = 9.2368, the difference of these quantities will be 12.5230 in the original drawing, or in the engraved plate, equal to one-third part, which makes the line V*q* equal one-third of the tang. of the angle of the abutment, to a radius 12.5230 = 8.831, scarcely differing from the figure in the engraved plate.

Fig. 9. is drawn to a radius of 10 inches, OV in the engraved

plate = 5 inches; which makes the line O*k* = OV $\frac{\text{sin. } 8° 49' 9''}{\text{sin. } 41° 10' 51''}$ = 1.1642 whence the line V*k* is equal to the tang. of 41° 10′ 51″, when the radius 6.1642 = 5.3926, which is nearly the length in inches of the line V*k* in the engraved plate.

On the Use of Logarithms, applied to the Computation of the subjoined Tables.

Logarithms are useful in making computations on mathematical subjects, particularly those that require the multiplication or division of quantities, by which the troublesome operations of multiplication and division are performed by corresponding additions and subtractions of logarithms only. By the preceding propositions it appears, that the quantity most frequently occurring in these computations is the weight of the first section, represented by *w*, and the cotang. of the angle of the first section. In the Table No. I. (Model No. 1.) Fig. 11, the angle of the first section A° = 5°, and in Table No. IX. Model No. 2, Fig. 13, the angle of the first section A° = 2° 38′ 0″; in the two Models which have been described, the weights of the first section in each Model are equal, each being the weight of a cubic inch and half of brass; the specific gravity of brass is to that of rain water in a proportion not very different from that of 8 to 1; sometimes a little exceeding, or sometimes a falling short of that proportion; on an average, therefore, the specific gravity of brass may be taken to that of water as 8 to 1: a cubic foot is equal in capacity 1728 cubic inches, and as a cubic foot of rain water has been found by experiment to weigh 1000 ounces avoirdupois almost exactly, it is evident, that the weight of a cubic inch of brass, of average specific gravity, weighs nearly

8 × .57870 = 4.62960 ounces, therefore $1\frac{1}{2}$ cubic inch of brass, weighs 6.9444 ounces, = .434027 parts of an avoirdupois pound $= w$;* the logarithm of which, or L. $w = 9.6375176$.

One of the most troublesome operations in the computation of the Tables subjoined, is to ascertain the weight of a single section, from having given the conditions on which the weight depends, which are as follows: The weight of one of the first or highest sections of the semiarch; the angle of the given section, with the angle of the abutment thereof, together with the angle of the abutment of the section preceding: to exemplify this rule, let it be proposed to find the weight of the section P in an arch of equilibration, in Table No. I. the first section of which = 5°, the angle of the section given = 5°, the angle of the abutment of $V^p = 75°$, the angle of the abutment preceding or $V^o = 70°$.

Computation for the weight in avoirdupois lbs.	Computation for L. w.
Log. w - $= 9.6375176$	Log. $\frac{1000}{1728} = 9.7624563$
L. cotang. $5° = 1.0580482$	L. $\frac{8}{16} = 9.6989700$
L. sin. $5° = 8.9402960$	L. $\frac{3}{2} = 0.1760913$
L. sec. $75° = 0.5870038$	
L. sec. $70° = 0.4659483$	L. $w = 9.6375176$
L. weight of P = 0.6888139	
Weight of P = 4.8844 lbs. avoirdupois.	

* In the Model No. 1. the dimensions of the first section of the semiarch are as follow: the base = 1 inch, the slant heighton either side = .961, and the breadth = 1.084; which makes the area of the first section parallel to the plane of the arch = 1 square inch; this multiplied into the depth or thickness, makes the solid contents of the first section = 1 × 1 × $1\frac{1}{2}$, which is a cubic inch and half a cubic inch.

In Model No. 2. the dimensions in the first section of the semiarch: the base, or the chord of 2° 38′ 0″, to a radius of 21.7598 = 1 inch, the slant height are as follows: the area of the first section parallel to the plane of the arch = 1 square inch; this multiplied into the deprh or thickness, which is $1\frac{1}{2}$ inches, the solid contents of the first section becomes = 1 × 1 × $1\frac{1}{2}$, or the solid contents of the first section = $1\frac{1}{2}$ cubic inches = .9782, and the breadth = 1.0449, which makes the solid contents of the section = $1\frac{1}{2}$ cubic inches, the weight of which = 4.3027 parts of an avoirdupois pound.

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By this means, another method of computing the weight of any section P is obtained, by putting the sum of the weights of all the sections from the summit to the section P; that is, the sum of all the weights from A° to $P^{\circ} = Sp$, and the sum of the weights of all the sections from A to O = So, the weight of the section P will be $= Sp - So$, for the rule in page 10,

Computation for Sp.

Log. w - $= 9.6375176$
L. cotang. $5^{\circ} = 1.0580482$
L. tang. $75^{\circ} = 0.5719475$

L. $Sp = 1.2675133$ $Sp = 18.514$

Computation for So.

Log. w - $= 9.6375176$
L. cotang. $5^{\circ} = 1.0580482$
L. tang. $70^{\circ} = 0.4389341$

L. $So = 1.1344999$ $So = 13.630$

$Sp = 18.514$
$So = 13.630$

$Sp - So$ = weight of the section P = 4.884, as before determined.

The computations of the dimensions (Fig. 7) of the brass sections in the Model No. 1. are much facilitated by the use of logarithms, particularly in finding the slant height Ot from the centre O of any section (K,) and the height of the section itself, or $St = Tt$.

Computation of the slant Height OT *of the Section* K.

It is first necessary to ascertain the area of the surface OST comprehended between the radii OS, OT, and the chord ST.

Since the radius OS = 11.46281 and the angle SOT = 5°, half SOT = 2° 30′ 0″, the

Sin. of 2° 30′ 0″ or $s = 8.6396796$ Log. $r = 1.0592910$
Cos. 2 30 0 or $c = 9.9995865$ 2

L. $sc = 8.6392661$ L. $r^2 = 2.1185820$
L. $\frac{1}{sc} = 1.3607339$ L. $sc = 8.6392661$

Log. of the area OST, or L. $sc \times r^2 = 0.7578481$

The area OST = 5.72595

The weight K = 2.19175

$K + r^2 sc =$ 7.91770

$L. \overline{K + r^2 sc} =$ 0.8985990

$L. \frac{1}{sc} =$ 1.3607339

$L. \frac{K + r^2 sc}{sc} =$ 2.2593329

$L. \sqrt{\frac{K + r^2 sc}{sc}} =$ 1.1296664

$Ot =$ 13.47928

Radius OS, or $= r =$ 11.46281

Height of the section $K = tt =$ 2.01647

Similar Computation for the Section L.

$L. r^2 =$ 2.1185820

$L. sc =$ 8.6392661

Log. of the area OTV = 0.7578481

area OTV = 5.72595

L = 2.70196

$L + r^2 sc =$ 8.42791

$L. \overline{L + r^2 sc} =$ 0.9257199

$L. \frac{1}{sc} =$ 1.3607339

$L. \frac{L + r^2 sc}{L\ sc} =$ 2.2864538

$L. \sqrt{\frac{L + r^2 sc}{sc}} =$ 1.1432269

$Ov =$ 13.90679

$r =$ 11.46281

Height of the section $L = vv =$ 2.44398

For the Section M.

$$L.\ r^2 = 2.1185820$$
$$L.\ sc = 8.6392661$$

$$\text{Log. of the area OVU} = 0.7578481$$
$$\text{area OVU} = 5.72595$$
$$M = 3.47366$$

$$M + r^2 sc = 9.19961$$
$$L.\ \overline{M + r^2 sc} = 0.9637694$$
$$L.\ \frac{1}{sc} = 1.3607339$$

$$L.\ \frac{M + r^2 sc}{sc} = 2.3245033$$
$$L.\ \sqrt{\frac{M + r^2 sc}{sc}} = 1.1622516$$
$$Ou = 14.52953$$
$$r = 11.46281$$

$$\text{Height of the section M} = uu = 3.06672$$

Computation of $\sqrt{\frac{2w + ab \times \sin. L^\circ}{\sin. L^\circ}}$.

$$L.\ a = 1.0827423$$
$$L.\ b = 1.0941566$$
$$L.\ \sin. L^\circ = 8.8806960$$

$$L.\ ab \times \sin. L^\circ = 1.0575949$$
$$ab \times \sin. L^\circ = 11.41812$$
$$2w = 7.$$

$$L.\ 2w + ab \times \sin. L^\circ = 18.41812$$

$$\text{L. } \overline{2w + ab} \times \text{sin. L}^\circ = 1.2652453$$

$$\text{L. sin. L}^\circ = 8.8806960$$

$$\text{L. } \frac{\overline{2w + ab} \times \text{sin. L}^\circ}{\text{sin. L}^\circ} = 2.3845493$$

$$\text{L. } \sqrt{\frac{\overline{2w + ab} \times \text{sin. L}^\circ}{\text{sin. L}^\circ}} = 1.1922746$$

$$\sqrt{\frac{\overline{2w + ab} \times \text{sin. L}^\circ}{\text{sin. L}^\circ}} = 15.56949$$

See page 19 and page 29, in which the computation is inserted of the quantity $w = \sqrt{\frac{\overline{2w + ab} \times \text{sin. R}^\circ}{\text{sin. R}^\circ}}$.

Computation for M°.

$$\text{L. } a = 1.1263101$$

$$\text{L. } b = 1.1375598$$

$$\text{L. sin. M}^\circ = 8.8391355$$

$$\text{L. } ab \times \text{sin. M}^\circ = 1.1030054$$

$$ab \times \text{sin. M}^\circ = 12.67667$$

$$2w = 7.5$$

$$\overline{2w + ab} \times \text{sin. M}^\circ = 20.17667$$

$$\text{L. } \overline{2w + ab} \times \text{sin. M}^\circ = 1.3048496$$

$$\text{L. sin. M}^\circ = 8.8391355$$

$$\text{L. } \frac{\overline{2w + ab} \times \text{sin. M}^\circ}{\text{sin. M}^\circ} = 2.4657141$$

$$\text{L. } \sqrt{\frac{\overline{2w + ab} \times \text{sin. M}^\circ}{\text{sin. M}^\circ}} = 1.2328570$$

$$\sqrt{\frac{\overline{2w + ab} \times \text{sin. M}^\circ}{\text{sin. M}^\circ}} = 17.0945$$

Breadth of the Section L.

Log. slant height from the centre = 1.1922561
L. 2 = 0.3010300
L. sin. $\frac{1}{2}$ L° = 8.5799524
Log. breadth of L° = 0.0732385
Breadth of L° = 1.1836
Breadth of L in the drawing = 1.1838
2 error.

Breadth of M.

Log. slant height from the centre = 1.2328570
L. 2 = 0.3010300
L. sin. $\frac{1}{2}$ M° = 8.5385170
L. breadth of M° = 0.0724040
Breadth of M° = 1.1814
Breadth of M by the drawing = 1.1814

Explanatory Notes on the Propositions in Pages 13 and 14 in the First Part of this Tract, in which A° = 5°, B° = 5°, = C° = D°, *&c. according to the Explanation in Page 12. The initial Pressure* $= \frac{1}{2 \times \sin. \frac{1}{2} A°}$, *or putting* $w = 1$, *the initial Pressure or* $p = \frac{1}{2} \times$ *cosecant* 2° 30′ 0″.

L. p = 1.0592904
L. cos. A° = 9.9983442
L. p × cos. A° = 1.0576346
p × cos. A° = 11.41917
V^a × sin. 2° 30′ 0″ = .04362
11.46279 = p

L. p = 1.0592904
L. sin. A° = 8.9402960
L. tang. 2° 30′ 0″ = 8.6400931
L. p × sin. A° × tang. 2° 30′ 0″ = 8.6396795
p × sin. A° × tang. 2 30 0 = .04362

It appears from this computation that p × sin. A° × tang. V^a is equal a × sin. V^a, when the weight of the first section, or $a = 1$.

The Weight and Pressure on the lowest Surface of the Section B.

$$\text{L.}\,p = 1.0592904$$
$$\text{L. cos. B}^\circ = 9.9983442$$
$$\text{L.}\,p \times \text{cos. B}^\circ = 1.0576346$$
$$p \times \text{cos. B}^\circ = 11.41917$$
$$b \times \text{sin. V}^b = .13153$$
$$11.55070 = q$$

$$\text{L.}\,p = 1.0592904$$
$$\text{L. sin. B}^\circ = 8.9402960$$
$$\text{L. sec. V}^b = 0.0037314$$
$$\text{L.}\,p \times \text{sin.B}^\circ \times \text{sec.V}^b = 0.0033178$$
$$p \times \text{sin.B}^\circ \times \text{sec.V}^b = 1.0076$$

$$\text{L.}\,p = 1.0592904$$
$$\text{L. sin. B}^\circ = 8.9402960$$
$$\text{L. tang. V}^b = 9.1194291$$
$$\text{L.}\,p \times \text{sin. B}^\circ \times \text{tang.V}^b = 9.1190155$$
$$p \times \text{sin. B}^\circ \times \text{tang.V}^b = .13153$$
$$p \times \text{cos. B}^\circ = 11.41917$$
$$p \times \text{cos. B}^\circ + p \times \text{sin. B}^\circ \times \text{tang.V}^b = 11.55070$$

ERRATA.

Page 5, line 5, *for* cotang. A × sec. A, *read* cotang. A° × sec. A°.
—— 6, — 19, *for* that part of weight, *read* that part of the weight.
—— 10, — 20, *for* $p = w \times$ cotang. A° × sec. *read* $w \times$ cotang. A° × sec. V^b.
—— 14, — 12, *for* area K*ts*, *read* T*tts*.
—— 14, — 17, *for* $x^2 - r^2 sc = k$, *read* $x^2 sc - r^2 sc = k$.
—— 23, — 5, *for* Fig. 9, *read* Fig. 8.
—— 24, — 1, *for* $x \times$ cotang. B, *read* $y \times$ cotang. B.
—— 24, — 5, *for* $x \times$ cotang. B°, *read* $z \times$ cotang. B°.
—— 24, — 16, *for* in No. 2, *read* in the Model No. 2.
—— 28, — 9, *for* OI, *read* OV.
—— 28, — 12, *for* the point II, *read* through the points II.
In Table No. IV. in the weight of the section I, *insert* 0.654983.
In Table No. X. *for* OV^2, *read* OV taken at 21.7598.

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TABLE No. I.

Shewing the weights of the several sections or wedges which form an arch of equilibration, when the angle of each section is 5°; and the weight of the highest wedge is assumed = 1. Also shewing the pressures on the lowest surface of each section, considered as an abutment.

The weights of the two first sections A in each semiarch = 1. The lateral or horizontal pressure = $p' = 11.430052$, Sd = the sum of the four successive weights = A + B + C + D, &c. &c.

Sections.	Angles of Sections.	Angles of the Abutments.		Weights of each Section.	Weights of the Semiarches.	Entire Pressures on the lowest Surface of each Section.
A	5°	V^a	5°	1.00000	$S\,a = 1.000000$	$11.47371 = p$
B	5	V^b	10	1.01542	$S\,b = 2.015426$	$11.60638 = q$
C	5	V^c	15	1.04724	$S\,c = 3.062673$	11 83327 $= r$
D	5	V^d	20	1.09752	$S\,d = 4.160196$	$12.16360 = s$
E	5	V^e	25	1.16972	$S\,e = 5.329920$	$12.61165 = t$
F	5	V^f	30	1.26922	$S\,f = 6.599144$	$13.19829 = v$
G	5	V^g	35	1.40427	$S\,g = 8.003420$	$13.95351 = u$
H	5	V^h	40	1.58754	$S\,h = 9.590960$	$14.92087 = w$
I	5	V^i	45	1.83910	$S\,i = 11.43006$	$16.16453 = x$
K	5	V^k	50	2.19175	$S\,k = 13.62181$	$17.78200 = y$
L	5	V^l	55	2.70196	$S\,l = 16.32377$	$19.92768 = z$
M	5	V^m	60	3.47366	$S\,m = 19.79743$	$22.86010 = a$
N	5	V^n	65	4.71440	$S\,n = 24.51183$	$27.04880 = b$
O	5	V^o	70	6.89199	$S\,o = 31.40382$	33 41923 $= c$
P	5	V^p	75	11.2537	$S\,p = 42.65753$	$44.16234 = d$
Q	5	V^q	80	22.1655	$S\,q = 64.82305$	$65.82304 = e$
R̂	5	V^r	85	65.8171	$S\,r = 130.6401$	$131.1450 = f$

Table No. II.

In which the angles of the sections are inferred from the given weights thereof, by the rule demonstrated in page 27 of the First Part of this Tract, and proport nal o the versed sines of a circle terminated by a horizontal line. The angle of the first section $A^\circ = 5^\circ$, and the initial pressure parallel to the horizon $= 11.43005$; a the pressure on the lowest surface of the first section $= 11.4371$.

Sections.	Weights of the sections.	Tang. of the angles of the sections.	Angles of the sections.	Angles of the abutments.	Pressures on the lowest surface of each section.
					Cotang. $A^\circ = 11.43005 = p'$
A $a =$	1.00000	$\frac{a \times \cos. V^o}{p' + a \times \sin. V^o} = \text{tang. } A^\circ$	5° 0′ 0″	$V^a =$ 5° 0′ 0″	$p =$ 11.47371
B $b =$	1.38053	$\frac{b \times \cos. V^a}{p + b \times \sin. V^a} = \text{tang. } B^\circ$	6° 45′ 53″	$V^b =$ 11° 45′ 53″	$q =$ 11.67533
C $c =$	2.51922	$\frac{c \times \cos. V^b}{q + c \times \sin. V^b} = \text{tang. } C^\circ$	11° 26′ 19″	$V^c =$ 23° 12′ 12″	$r =$ 12.43599
D $d =$	4.40742	$\frac{d \times \cos. V^c}{r + d \times \sin. V^c} = \text{tang. } D^\circ$	15° 57′ 5″	$V^d =$ 39° 9′ 17″	$s =$ 14.74004
E $e =$	7.03074	$\frac{e \times \cos. V^d}{s + e \times \sin. V^d} = \text{tang. } E^\circ$	15° 52′ 6″	$V^e =$ 55° 1′ 23″	$t =$ 19.93919
F $f =$	10.36922	$\frac{f \times \cos. V^e}{t + f \times \sin. V^e} = \text{tang. } F^\circ$	11° 48′ 25″	$V^f =$ 66° 49′ 48″	$v =$ 29.05015
G $g =$	14.39746	$\frac{g \times \cos. V^f}{v + g \times \sin. V^f} = \text{tang. } G^\circ$	7° 37′ 48″	$V^g =$ 74° 27′ 36″	$u =$ 42.66403
H $h =$	19.80480	$\frac{h \times \cos. V^g}{u + h \times \sin V^g} = \text{tang. } H^\circ$	4° 47′ 14″	$V^h =$ 79° 14′ 50″	$w =$ 61.26446
I $i =$	24.39556	$\frac{i \times \cos. V^h}{w + i \times \sin. V^h} = \text{tang. } I^\circ$	3° 3′ 24″	$V^i =$ 82° 18′ 14″	$x =$ 85.35309
K $k =$	30.28932	$\frac{k \times \cos. V^i}{x + k \times \sin. V^i} = \text{tang. } K^\circ$	2° 1′ 5″	$V^k =$ 84° 19′ 19″	$y =$ 115.44084

Table No. III.

In which the angles of the sections are 1°, 2°, 3°, &c. making the angles of the abutments 1°, 3°, 6°, 10°, for inferring the weights of the successive sections and the sums thereof, with the pressures on the lowest surface of each section, as computed from the general rules in page 15, as they are inserted in the 5th, 6th, and 7th columns of this Table.

	Angles of the sections.	Angles between the lowest surface of each section and the vertical, or angles of the abutments.		Weights of the successive sections.	Weights of the successive semiarches.	Pressures on the abutments.
A	1°	V^a	1°	1.000000	1.00000	$p = 57.29869$
B	2	V^b	3	2.002440	3.00244	$q = 57.36859$
C	3	V^c	6	3.018978	6.02141	$r = 57.60538$
D	4	V^d	10	4.080347	10.10176	$s = 58.17374$
E	5	V^e	15	5.249031	15.35079	$t = 59.31090$
F	6	V^f	21	6.640753	21.99154	$v = 61.36580$
G	7	V^g	28	8.470050	30.46159	$u = 64.88482$
H	8	V^b	36	11.16197	41.62356	$w = 70.81421$
I	9	V^i	45	15.66635	57.28991	$x = 81.02014$
K	10	V^k	55	24.52854	81.81845	$y = 99.88185$
L	11	V^l	66	46.85674	128.6751	$z = 140.8525$
M	12	V^m	78	140.8525	269.5276	$a = 275.5490$

Table No. IV.

In this Table the angle of the first section $A^{\circ} = 5^{\circ}$, and the angles B°, C°, D°, &c. are assumed of any given magnitude, taken at hazard $= 6^{\circ}$, 8°, 12°, &c. making the angles of the abutments $= 5^{\circ}$, 11°, 19°, 31°, and $p = 11.4737$, &c. The initial pressure $p' = 11{,}43005$.

Sections.	Angles of the sections	Angles contained between the lower surface of each section and the vertical line.	Weights of the sections.	Weights of the semi-arches, found by calculating from the values inserted in page 14 of the Dissertation on Arches.	Entire pressures on the lower surface of each section, considered as an abutment, found by calculations from the values for the pressures inserted in page 14 of the Dissertation on Arches.
A	5°	$V^a = 5^{\circ}$	$a = 1.000000$	1.00000	$11.47371 = p$
B	6	$V^b = 11$	$b = 1.221776$	2.22177	$11.64392 = q$
C	8	$V^c = 19$	$c = 1.713895$	3.93567	$12.08864 = r$
D	12	$V^d = 31$	$d = 2.932180$	6.86785	$13.33465 = s$
E	10	$V^e = 41$	$e = 3.068117$	9.93596	$15.14492 = t$
F	9	$V^f = 50$	$f = 3.685800$	13.62176	$17.78193 = v$
G	4	$V^g = 54$	$g = 2.110300$	15.73206	$19.44585 = u$
H	2	$V^h = 56$	$h = 1.213626$	16.94569	$20.44014 = w$
I	1	$V^i = 57$	$i = .654983$	17.60067	$20.98633 = x$
K	7	$V^k = 64$	$k = 5.834303$	23.43498	$26.07373 = y$
L	4	$V^l = 68$	$l = 4.855258$	28.29023	$30.51193 = z$
M	3	$V^m = 71$	$m = 4.904875$	33.19511	$35.10776 = a$
N	5	$V^n = 76$	$n = 12.64806$	45.84317	$47.24652 = b$
O	12	$V^o = 88$	$o = 281.4682$	327.3113	$327.5108 = c$
P	1	$V^p = 89$	$p = 327.5107$	654.8220	$654.9206 = d$

Table V.

Shewing the angles of the wedges in an arch of equilibration, in which the weights of the several sections are $= 1$, the angle of the first section $= 15°$; the initial pressure parallel to the horizon $p' = 3{\cdot}73205$, and the pressure on the lowest surface of the first section $= p = 3.86370$.

Sections.		Weights of the sections.	Tang. of the angles of the sections.	Angles of the sections.	Angles of the abutments.	Pressures on the lowest surface of each section.
A	$a =$	1	$\frac{a \times \cos. V^{\circ}}{p' + a \times \sin. V^{\circ}} = \text{tang. } A^{\circ} =$	15° 0′ 0″	$V^{a} = 15° \ 0' \ 0''$	Cosec. $A = p = 3{\cdot}86370$
B	$b =$	1	$\frac{b \times \cos. V^{a}}{p + b \times \sin. V^{a}} = \text{tang. } B^{\circ} =$	13° 11′ 12″	$V^{b} = 28° \ 11' \ 12''$	$4{\cdot}234170 = q$
C	$c =$	1	$\frac{c \times \cos. V^{b}}{q + c \times \sin. V^{d}} = \text{tang. } C^{\circ} =$	10° 36′ 25″	$V^{c} = 38° \ 47' \ 37''$	$4{\cdot}788337 = r$
D	$d =$	1	$\frac{d \times \cos. V^{c}}{r + d \times \sin. V^{d}} = \text{tang. } D^{\circ} =$	8° 11′ 27″	$V^{d} = 46° \ 59' \ 4''$	$5{\cdot}470659 = s$
E	$e =$	1	$\frac{e \times \cos. V^{d}}{s + e \times \sin. V^{e}} = \text{tang. } E^{\circ} =$	6° 16′ 38″	$V^{e} = 53° \ 15' \ 42''$	$6.239237 = t$
F	$f =$	1	$\frac{f \times \cos. V^{e}}{t + f \times \sin. V^{f}} = \text{tang. } F^{\circ} =$	4° 51′ 22″	$V^{f} = 58° \ 7' \ 4''$	$7{\cdot}065979 = v$
G	$g =$	1	$\frac{g \times \cos. V^{f}}{v + g \times \sin. V^{g}} = \text{tang. } G^{\circ} =$	3° 49′ 3″	$V^{g} = 61° \ 56' \ 7''$	$7{\cdot}932716 = u$
H	$h =$	1	$\frac{h \times \cos. V^{h}}{u + h \times \sin. V^{h}} = \text{tang. } H^{\circ} =$	3° 3′ 18″	$V^{h} = 64° \ 59' \ 25''$	$8.827677 = w$
I	$i =$	1	$\frac{i \times \cos. V^{i}}{w + i \times \sin. V^{i}} = \text{tang. } I^{\circ} =$	2° 29′ 53″	$V^{i} = 67° \ 28' \ 18''$	$9{\cdot}743980 = x$

Table No. VI.

Shewing the angles of the several sections, in which the weight of each of the sections = 1, and the angle of the two highest sections = A°; in each semiarch = 5°, the initial horizontal pressure = cotang. 5° = 11.43005; and therefore the pressure on the lowest surface of the first section = cosec. 5° = 11.47371.

Sections.		Weights of the sections.	Tang. of the angles of the sections.	Angles of the sections.	Angles of the abutments.	Pressures on the lowest surface of each section.
A	$a =$	1	$\frac{a \times \cos. V^o}{p' + a \times \sin. V^o} = \text{tang. } A^\circ =$	5° 0′ 0″	$V^a =$ 5° 0′ 0″	$p =$ 11.47371
B	$b =$	1	$\frac{b \times \cos. V^a}{p + b \times \sin. V^a} = \text{tang. } B^\circ =$	4° 55′ 30″	$V^b =$ 9° 55′ 30″	$q =$ 11.60380
C	$c =$	1	$\frac{c \times \cos. V^b}{q + c \times \sin. V^b} = \text{tang. } C^\circ =$	4° 46′ 53″	$V^c =$ 14° 42′ 23″	$r =$ 11.81728
D	$d =$	1	$\frac{d \times \cos. V^c}{r + d \times \sin. V^c} = \text{tang. } D^\circ =$	4° 34′ 52″	$V^d =$ 19° 17′ 15″	$s =$ 12.10992
E	$e =$	1	$\frac{e \times \cos. V^d}{s + e \times \sin. V^d} = \text{tang. } E^\circ =$	4° 20′ 20″	$V^e =$ 23° 37′ 35″	$t =$ 12.47598
F	$f =$	1	$\frac{f \times \cos. V^e}{t + f \times \sin. V^e} = \text{tang. } F^\circ =$	4° 4′ 11″	$V^f =$ 27° 41′ 46″	$v =$ 12.90929
G	$g =$	1	$\frac{g \times \cos. V^f}{v + g \times \sin. V^f} = \text{tang. } G^\circ =$	3° 47′ 16″	$V^g =$ 31° 29′ 2″	$u =$ 13.40333
H	$h =$	1	$\frac{h \times \cos. V^g}{u + h \times \sin. V^g} = \text{tang. } H^\circ =$	3° 30′ 15″	$V^h =$ 34° 59′ 17″	$w =$ 13.95167
I	$i =$	1	$\frac{i \times \cos. V^h}{w + i \times \sin. V^h} = \text{tang. } I^\circ =$	3° 13′ 42″	$V^i =$ 38° 12′ 59″	$x =$ 14.54815
K	$k =$	1	$\frac{k \times \cos. V^i}{x + k \times \sin. V^i} = \text{tang. } K^\circ =$	2° 57′ 52″	$V^k =$ 41° 10′ 51″	$y =$ 15.18711
L	$l =$	1	$\frac{l \times \cos. V^k}{y + l \times \sin. V^k} = \text{tang. } L^\circ =$	2° 43′ 10″	$V^l =$ 43° 54′ 1″	$z =$ 15.86340

Table No. VII.

Containing the weights in an arch of equilibration, in which the angles of each section are = 2° 30′ 0″, the pressure on the lowest surface of each section; the initial pressure parallel to the horizon = cotang. 2° 30′ = 22.90376 = p'; and the pressure on lowest surface of the first section = cosec. 2° 30′ = 22.92558.

Sections	Angles of the sections.	Angles of the abutments.	Weights of the sections.	Sums of the weights of the sections.	Pressures on the lowest surface of each section.
A	2° 30′	V^a = 2° 30′	1.00000	S*a* = 1.000000	22.92558 = *p*
B	2 30	V^b = 5 0	1.00382	S*b* = 2.003820	22.99125 = *q*
C	2 30	V^c = 7 30	1.01151	S*c* = 3.015331	23.10140 = *r*
D	2 30	V^d = 10 0	1.02322	S*d* = 4.038552	23.25714 = *s*
E	2 30	V^e = 12 30	1.03909	S*e* = 5.077642	23.45986 = *t*
F	2 30	V^f = 15 0	1.05940	S*f* = 6.137047	23.71172 = *v*
G	2 30	V^g = 17 30	1.09448	S*g* = 7.221530	24.01526 = *u*
H	2 30	V^h = 20 0	1.11476	S*h* = 8.336290	24.37368 = *w*
I	2 30	V^i = 22 30	1.15076	S*i* = 9.487050	24.79086 = *x*
K	2 30	V^k = 25 0	1.19315	S*k* = 10.68020	25.27151 = *y*
L	2 30	V^l = 27 30	1.24374	S*l* = 11.92394	25.82129 = *z*
M	2 30	V^m = 30 0	1.29956	S*m* = 13.22350	26.44699 = *a*
N	2 30	V^n = 32 30	1.36780	S*n* = 14.59130	27.15674 = *b*
O	2 30	V^o = 35 0	1.44608	S*o* = 16.03738	27.96033 = *c*
P	2 30	V^p = 37 30	1.53730	S*p* = 17.57468	28.86956 = *d*
Q	2 30	V^q = 40 0	1.64386	S*q* = 19.21854	29.89874 = *e*
R	2 30	V^r = 42 30	1.76889	S*r* = 20.98743	31.06533 = *f*
S	2 30	V^s = 45 0	1.91634	S*s* = 22.90377	32.39081 = *g*
T	2 30	V^t = 47 30	2.09130	S*t* = 24.99507	33.90187 = *h*
V	2 30	V^v = 50 0	2.30058	S*v* = 27.29565	35.63193 = *i*
U	2 30	V^u = 52 30	2.55312	S*u* = 29.84877	37.62355 = *k*
W	2 30	V^w = 55 0	2.85112	S*w* = 32.70989	39.93149 = *l*
X	2 30	V^x = 57 30	3.25182	S*x* = 35.95171	42.62755 = *m*
Y	2 30	V^y = 60 0	3.71877	S*y* = 39.67048	45.80753 = *n*
Z	2 30	V^z = 62 30	4.32724	S*z* = 43.99772	49.60224 = *o*
A	2 30	V^a = 65 0	5.11958	S*a* = 49.11730	54.19492 = *p*
B	2 30	V^b = 67 30	6.17727	S*b* = 55.29457	59.85041 = *q*
C	2 30	V^c = 70 0	7.63300	S*c* = 62.92757	66.96511 = *r*
D	2 30	V^d = 72 30	9.71389	S*d* = 72.64146	74.11813 = *s*
E	2 30	V^e = 75 0	12.83654	S*e* = 85.47800	88.49336 = *t*

TABLE No. VIII.

Shewing the angles of fifty sections, forming an arch of equilibration, calculated from given weights of the sections when the angle of the first section is one degree = A°; and the weight thereof is denoted by unity; the weights of the successive sections encreasing by equal differences from 1 to 3, which is the weight of the twenty-fifth section = Z in each semiarch. The initial pressure parallel to the horizon p' = cotang. A° = 57.28996: the pressure on the lowest surface of the first section is = p = 57.29868 = cosecant A°.

Sections.		Weights of the sections.	Tang. of the angles of the sections.	Angles of the sections.	Angles of the abutments.	Pressures on the lowest surface of each section.
A	a =	1.000000	$\frac{a \times \cos. V^{o}}{p' + a \times \sin. V^{o}}$ = tang. A° =	1° 0′ 0″	V^{a} = 1° 0′ 0″	p = 57.29868
B	b =	1.083333	$\frac{b \times \cos. V^{a}}{p + b \times \sin. V^{a}}$ = tang. B° =	1° 4′ 57″,457	V^{b} = 2° 4′ 57″,457	q = 57.32782
C	c =	1.166666	$\frac{c \times \cos. V^{b}}{q + c \times \sin. V^{b}}$ = tang. C° =	1° 9′ 51″,204	V^{c} = 3° 14′ 48″,661	r = 57.38205
D	d =	1.250000	$\frac{d \times \cos. V^{c}}{r + d \times \sin. V^{c}}$ = tang. D° =	1° 14′ 39″,795	V^{d} = 4° 29′ 28″,456	s = 57.46639
E	e =	1.333333	$\frac{e \times \cos. V^{d}}{s + e \times \sin. V^{d}}$ = tang. E° =	1° 19′ 21″,558	V^{e} = 5° 48′ 50″,014	t = 57.58614
F	f =	1.416666	$\frac{f \times \cos. V^{e}}{t + f \times \sin. V^{e}}$ = tang. F° =	1° 23′ 54″,634	V^{f} = 7° 12′ 44″,648	v = 57.74684
G	g =	1.500000	$\frac{g \times \cos. V^{f}}{v + g \times \sin. V^{f}}$ = tang. G° =	1° 28′ 16″,987	V^{g} = 8° 41° 1″,638	u = 57.95427
H	b =	1.583333	$\frac{h \times \cos. V^{g}}{u + h \times \sin. V^{g}}$ = tang. H° =	1° 32′ 26″,417	V^{h} = 10° 13′ 28″,055	w = 58.21435
I	i =	1.666666	$\frac{i \times \cos. V^{h}}{w + i \times \sin. V^{h}}$ = tang. I° =	1° 36′ 20″,646	V^{i} = 11° 49′ 48″,701	x = 58.53326
K	k =	1.750000	$\frac{k \times \cos. V^{i}}{x + k \times \sin. V^{i}}$ = tang. K° =	1° 39′ 57″,365	V^{k} = 13° 29′ 46″,066	y = 58.91692
L	l =	1.833333	$\frac{l \times \cos. V^{k}}{y + l \times \sin. V^{k}}$ = tang. L° =	1° 43′ 14″,297	V^{l} = 15° 13′ 0″,363	z = 59.37154
M	m =	1.916666	$\frac{m \times \cos. V^{l}}{z + m \times \cos. V^{l}}$ = tang. M° =	1° 46′ 9″,294	V^{m} = 16° 59′ 9″,667	a = 59.90315
N	n =	2.000000	$\frac{n \times \cos. V^{m}}{a + n \times \sin. V^{m}}$ = tang. N° =	1° 48′ 40″,404	V^{n} = 18° 47′ 50″,071	b = 60.51760
O	o =	2.083333	$\frac{o \times \cos. V^{n}}{b + o \times \sin. V^{n}}$ = tang. O° =	1° 50′ 45″,954	V^{o} = 20° 38′ 36″,071	c = 61.22067
P	p =	2.166666	$\frac{p \times \cos. V^{o}}{c' + p \times \sin. V^{o}}$ = tang. P° =	1° 52′ 24″,611	V^{p} = 22° 31′ 0″,715	d = 62.01767
Q	q =	2.250000	$\frac{q \times \cos. V^{p}}{d + q \times \sin. V^{p}}$ = tang. Q° =	1° 53′ 35″,611	V^{q} = 24° 24′ 36″,326	e = 62.91365
R	r =	2.333333	$\frac{r \times \cos. V^{q}}{e + r \times \sin. V^{q}}$ = tang. R° =	1° 54′ 18″,421	V^{r} = 26° 18′ 54″,747	f = 63.91325
S	s =	2.416666	$\frac{s \times \cos. V^{r}}{f + s \times \sin. V^{r}}$ = tang. S° =	1° 54′ 33″,186	V^{s} = 28° 13′ 27″,933	g = 65.02070
T	t =	2.500000	$\frac{t \times \cos. V^{s}}{g + t \times \sin. V^{s}}$ = tang. T° =	1° 54′ 20″,477	V^{t} = 30° 7′ 48″,410	b = 66.23967
U	u =	2.583333	$\frac{u \times \cos. V^{t}}{h + u \times \sin. V^{t}}$ = tang. U° =	1° 53′ 41″,334	V^{u} = 32° 1′ 29″,744	i = 67.57337
V	v =	2.666666	$\frac{v \times \cos. V^{u}}{i + v \times \sin. V^{u}}$ = tang. V° =	1° 52′ 37″,272	V^{v} = 33° 54′ 7″,016	k = 69.02449
W	w =	2.750000	$\frac{w \times \cos. V^{v}}{k + w + \sin. V^{v}}$ = tang. W° =	1° 51′ 10″,121	V^{w} = 34° 45′ 17″,137	l = 70.59525
X	x =	2.833333	$\frac{x \times \cos. V^{w}}{l + x \times \sin. V^{w}}$ = tang. X° =	1° 49′ 22″,000	V^{x} = 37° 34′ 39″,137	m = 72.28737
Y	y =	2.916666	$\frac{y \times \cos. V^{x}}{m + y \times \sin. V^{x}}$ = tang. Y° =	1° 47′ 15″,273	V^{y} = 39° 21′ 54″,410	n = 74.10210
Z	z =	3.000000	$\frac{z \times \cos. V^{y}}{\blacksquare + z \times \sin. V^{y}}$ = tang. Z° =	1° 44′ 52″,429	V^{z} = 41° 6′ 46″,839	o = 76.04024

TABLE No. IX.

Containing the angles of thirty-four sections or wedges, constituting the model of an arch, No. 2, the weights of which increase regularly in each semiarch, from 1, which is assumed as the weight of the first section, to 5, which is the weight of the lowest or seventeenth section from the summit: the angle of the first section $A^\circ = 2^\circ\ 38'\ 0''$, and B, C, D, &c. are inferred by the rule in page 15, from the weights of the said sections. The initial pressure parallel to the horizon $=$ cotang. $2^\circ\ 38' = 21.7425 = p'$: the pressure upon the lowest surface of the section A, cosecant $A^\circ = 21.76555 = p$.

Sections.	Weights of the sections.	Tang. of the angles of the sections.	Angles of the sections.	Angles of the abutments.	Pressures on the lowest surface of each section.
A $a =$	1.00	$\frac{a \times \cos. V^{o}}{p' + a \times \sin. V^{a}} = \text{tang. } A^\circ =$	$2^\circ\ 38'\ 0',$	$V^a = 2^\circ\ 38'\ 0''$	$p = 21.76555$
B $b =$	1.25	$\frac{b \times \cos. V^{a}}{p + b \times \sin. V^{a}} = \text{tang. } B^\circ =$	$3^\circ\ 16'\ 29''$	$V^b = 5^\circ\ 54'\ 29''$	$q = 21.85867$
C $c =$	1.50	$\frac{c \times \cos. V^{b}}{q + c \times \sin. V^{b}} = \text{tang. } C^\circ =$	$3^\circ\ 52'\ 39''$	$V^c = 9^\circ\ 47'\ 8''$	$r = 22.06356$
D $d =$	1.75	$\frac{d \times \cos. V^{c}}{r + d \times \sin. V^{c}} = \text{tang. } D^\circ =$	$4^\circ\ 24'\ 36''$	$V^d = 14^\circ\ 11'\ 44''$	$s = 22.42739$
E $e =$	2.00	$\frac{e \times \cos. V^{d}}{s + e \times \sin. V^{d}} = \text{tang. } E^\circ =$	$4^\circ\ 50'\ 9''$	$V^e = 19^\circ\ 1'\ 53''$	$t = 22.99972$
F $f =$	2.25	$\frac{f \times \cos. V^{e}}{t + f \times \sin. V^{e}} = \text{tang. } F^\circ =$	$5^\circ\ 7'\ 16''$	$V^f = 24^\circ\ 9'\ 9''$	$v = 23.82853$
G $g =$	2.50	$\frac{g \times \cos. V^{f}}{v + g \times \sin. V^{f}} = \text{tang. } G^\circ =$	$5^\circ\ 14'\ 41''$	$V^g = 29^\circ\ 23'\ 50''$	$u = 24.95590$
H $h =$	2.75	$\frac{h \times \cos. V^{g}}{u + h \times \sin. V^{g}} = \text{tang. } H^\circ =$	$5^\circ\ 12'\ 14''$	$V^h = 34^\circ\ 36'\ 4''$	$w = 26.41465$
I $i =$	3.00	$\frac{i \times \cos. V^{h}}{w + i \times \sin. V^{h}} = \text{tang. } I^\circ =$	$5^\circ\ 1'\ 8''$	$V^i = 39^\circ\ 37'\ 12''$	$x = 28.22645$
K $k =$	3.25	$\frac{k \times \cos. V^{i}}{x + k \times \sin. V^{i}} = \text{tang. } K^\circ =$	$4^\circ\ 43'\ 23''$	$V^k = 44^\circ\ 20'\ 35''$	$y = 30.40220$
L $l =$	3.50	$\frac{l \times \cos. V^{k}}{y + l \times \sin. V^{k}} = \text{tang. } L^\circ =$	$4^\circ\ 21'\ 27''$	$V^l = 48^\circ\ 42'\ 2''$	$z = 32.94376$
M $m =$	3.75	$\frac{m \times \cos. V^{l}}{z + m \times \sin. V^{l}} = \text{tang. } M^\circ =$	$3^\circ\ 57'\ 33''$	$V^m = 52^\circ\ 39'\ 35''$	$a = 35.84656$
N $n =$	4.00	$\frac{n \times \cos. V^{m}}{a + n \times \sin. V^{m}} = \text{tang. } N^\circ =$	$3^\circ\ 33'\ 26''$	$V^n = 56^\circ\ 13'\ 1''$	$b = 39.10209$
O $o =$	4.25	$\frac{o \times \cos. V^{n}}{b + o \times \sin. V^{n}} = \text{tang. } O^\circ =$	$3^\circ\ 10'\ 21''$	$V^o = 59^\circ\ 23'\ 22''$	$c = 42.69992$
P $p =$	4.50	$\frac{p \times \cos. V^{o}}{c + p \times \sin. V^{o}} = \text{tang. } P^\circ =$	$2^\circ\ 49'\ 0''$	$V^p = 62^\circ\ 12'\ 22''$	$d = 46.62917$
Q $q =$	4.75	$\frac{q \times \cos. V^{p}}{q + d \times \sin. V^{p}} = \text{tang. } Q^\circ =$	$2^\circ\ 29'\ 42''$	$V^q = 64^\circ\ 42'\ 4''$	$e = 50.87939$
R $r =$	5.00	$\frac{r \times \cos. V^{q}}{r + e \times \sin. V^{q}} = \text{tang. } R^\circ =$	$2^\circ\ 12'\ 31''$	$V^r = 66^\circ\ 54'\ 35''$	$f = 55.44104$

Table No. X.

Shewing the method of determining the points in the vertical line OV, from which lines being drawn to the several points B, C, D, E, &c. will determine the positions of the abutments on which the said sections are sustained: when the angle of the first section A° is assumed = 2° 38′, and the angles of the sections B°, C°, D°, &c. are inferred from the weights thereof. The distances OA, OB, OC, &c. being negative, shew that the numbers corresponding are to be subtracted from the radius OV.

	A	B	C	D	E	F	G
Angles of the abutments Angles at the centre	2° 38′ 0″ 2° 38′ 0″	5° 54′ 29″ 5° 16′ 0″	9° 47′ 8″ 7° 54′ 0″	14° 11′ 44″ 10° 32′ 0″	19° 1′ 53″ 13° 10′ 0″	24° 9′ 9″ 15° 48′ 0″	29° 23′ 50″ 18° 26′ 0″
Differences of the angles	0° 0′ 0″	0° 38′ 29″	1° 53′ 8″	3° 39′ 44″	5° 51′ 53″	8° 21′ 9″	10° 57′ 50″
Log. radius = 21.7598 inches Log. sin. differences of angles Log. cosec. of the angles of the abutments		1.3376550 8.0489897 0 9874480	1.3376550 8.5172383 0.7696505	1.3376550 8.8053263 0.6104225	1.3376550 9.0093662 0 4866677	1.3376550 9.1621545 0.3880997	1.3376550 9.2791883 0.3090410
Log. distances from the centre Distances from the centre	OA = − 0.0000	0.3740927 OB = − 2.3664	0.6245438 OC = − 4.2125	0.7534038 OD = − 5.6676	0.8336889 OE = − 6.8183	0.8879092 OF = − 7.7252	0.9258843 OG = − 8.4311

		H	I	K	L	M	N
Angles of the abutment Angles at the centre		34° 36′ 4″ 21° 4′ 0″	39° 37′ 12″ 23° 42′ 0″	44° 20′ 35″ 26° 20′ 0″	48° 42′ 2″ 28° 58′ 0″	52° 39′ 35″ 31° 36′ 0″	56° 13′ 1″ 34° 14′ 0″
Differences of the angles		13° 32′ 4″	15° 55′ 12″	18° 0′ 35″	19° 44′ 2″	21° 03′ 35″	21° 59′ 1″
Log. radius Log. sin. differences of angles Log. cosec. of the angles of the abutments		1.3376550 9.3692713 0.2457589	1.3376550 9.4382178 0.1953884	1.3376550 9.4892091 0.1554521	1.3376550 9.5284694 0.1242034	1.3376550 9.5555066 0.0996070	1.3376550 9.5732678 0.0803211
Log. distances from the centre Distances from the centre		0.9526852 OH = − 8.9677	0.9712612 OI = − 9.3597	0 9824162 OK = − 9.6032	0.9903278 OL = − 9 7797	0.9927686 OM = − 9.8348	0.9012439 ON = − 9.8084

		O	P	Q	R
Angles of the abutments Angles at the centre		59° 23′ 22″ 36° 52′ 0″	62° 12′ 22″ 39° 30′ 0″	64° 42′ 4″ 42° 8′ 0″	66° 54′ 35″ 44° 46′ 0″
Differences of the angles		22° 31′ 22″	22° 42′ 22″	22° 34′ 4″	22° 8′ 35″
Log. radius Log. sin. differences of angles Log. cosec. of the angles of the abutments		1.3376550 9.5832562 0.0651743	1.3376550 9.5865923 0.0532380	1.3376550 9.5840779 0.0437879	1.3376550 9.5762497 0.0362650
Log distances from the centre Distances from the centre		0.9860855 OO = − 9.6847	0.9774853 OP = − 9.4948	0.9655208 OQ = − 9.2368	0.9501697 OR = − 8.9160

Table No. XI.

Shewing the method of determining the points in the line OV, taken = 10 inches; from which, lines being drawn to the several points B, C, D, &c. will determine the position of the abutments on which the said sections are sustained when the angle of the

Log. radius - - - -	1.3376550	1.3376550	1.3376550	1.3376550
Log. sin. differences of angles -	9.5832562	9.5865923	9.5840779	9.5762497
Log. cosec. of the angles of the abutments	0.0651743	0.0532380	0.0437879	0.0362650
Log. distances from the centre -	0.9860855	0.0774853	0.9655208	0.9501697
Distances from the centre - -	OO = — 9.6847	OP = — [illegible]	OQ = — 9.2368	OR = — 8.9160

TABLE No. XI.

Shewing the method of determining the points in the line OV, taken = 10 inches; from which, lines being drawn to the several points B, C, D, &c. will determine the position of the abutments on which the said sections are sustained when the angle of the first section A is assumed = 5°, and the angles of the sections B°, C°, D°, &c. are inferred from the weights thereof, assumed = A = B = C = D, &c. = 1, as stated in Table VI.

		A	B	C	D	E	F
Angles at the centre - - - -	—	5° 0′ 0″	10° 0′ 0″	15° 0′ 0″	20° 0′ 0″	25° 0′ 0″	30° 0′ 0″
Angles of the abutments - - -	+	5° 0′ 0″	9° 55′ 30″	14° 42′ 23″	19° 17′ 15″	23° 37′ 35″	27° 41′ 46″
Difference of the angles - - - -	∓	0° 0′ 0″	0° 4′ 30″	0° 17′ 37″	0° 42′ 45″	1° 22′ 25″	2° 18′ 14″
Log. 10 inches - - - - - -	=	- - -	1.0000000	1.	1.	1.	1.
Log. sin. differences of the angles - -	=	- - -	7.1169385	7.7096480	8.0946510	8.3796996	8.6042219
Log. cosec. angles of the abutments - -		- - -	0.7635662	0.5953958	0.4810803	0.3971037	0.3327507
Log. distance from the centre - - -		- - -	8.8805047	9.3050438	9.5757313	9.7768033	9.9369726
Distances from the centre O - - -	=	- - -	OB = ∓ .075946	OC = ∓ .20185	OD = ∓ .37647	OE = ∓ .59814	OF = ∓ .86491
Radius added to the distances from O - -		- - -	10.075946	10.20185	10.37647	10.59814	10.86491
Log. distances from the centre - - -		- - -	1.00328	1.00868	1.01603	1.02522	1.03602
Log. tang. of the angles of the abutments -		- - -	9.24298	9.41907	9.54400	9.64091	9.72009
Log. tang. of the angle of the abutments to radius 10		- - -	0.24626	0.42775	0.56003	0.66613	0.75611
Tang. of the angle of the abutments to radius 10		- - -	Inches ∓ 1.7630	∓ 2.6776	∓ 3.6311	∓ 4.6359	∓ 5.7032

			G	H	I	K	L
Angles at the centre - - - - -	—	- - -	35° 0′ 0″	40° 0′ 0″	45° 0′ 0″	50° 0′ 0″	55° 0′ 0″
Angles of the abutments - - -	+	- - -	31° 29′ 2″	34° 59′ 17″	38° 12′ 59″	41° 10′ 51″	43° 54′ 1″
Difference of the angles - - - -	∓	- - -	3° 30′ 58″	5° 0′ 43″	6° 47′ 1″	8° 49′ 9″	11° 5′ 59″
Log. 10 inches - - - - - -	=	- - -	1.	1.	1.0000000	1.	1.
Log. sin. differences of the angles - -	=	- - -	8.7876673	8.9413296	9.0723232	9.1855886	9.2834696
Log. cosec. angles of the abutments - -		- - -	0.2821142	0.2415380	0.2085668	0.1814853	0.1590128
Log. distance from the centre - - -		- - -	0.0697815	0.1828676	0.2808900	0.3670739	0.4424824
Distances from the centre O - - -	=	- - -	OG = ∓ 1.1743	OH = ∓ 1.5235	OI = ∓ 1.9093	OK = ∓ 2.3285	OL = ∓ 2.7700
Radius added to the distances from O - -		- - -	11.1743	11.5235	11.9093	12.3285	12.7700
Log. distances from the centre - - -		- - -	1.04821	1.06156	1.07587	1.09089	1.10619
Log. tang. of the angles of the abutments -		- - -	9.78704	9.84503	9.89618	9.94193	9.98332
Log. tang. of the angle of the abutments to radius 10		- - -	0.83525	0.90659	0.97205	1.03282	1.08951
Tang. of the abutments to radius 10 - -		- - -	Inches ∓ 6.8432	∓ 8.0649	∓ 9.3769	∓ 10.785	∓ 12.289

When the angle of the abutment is greater than the angle at the centre, the upper sign prevails, as in Fig. 8; but when the angle at the abutment is less than the angle of the centre, the lower sign prevails, as in Fig. 9.

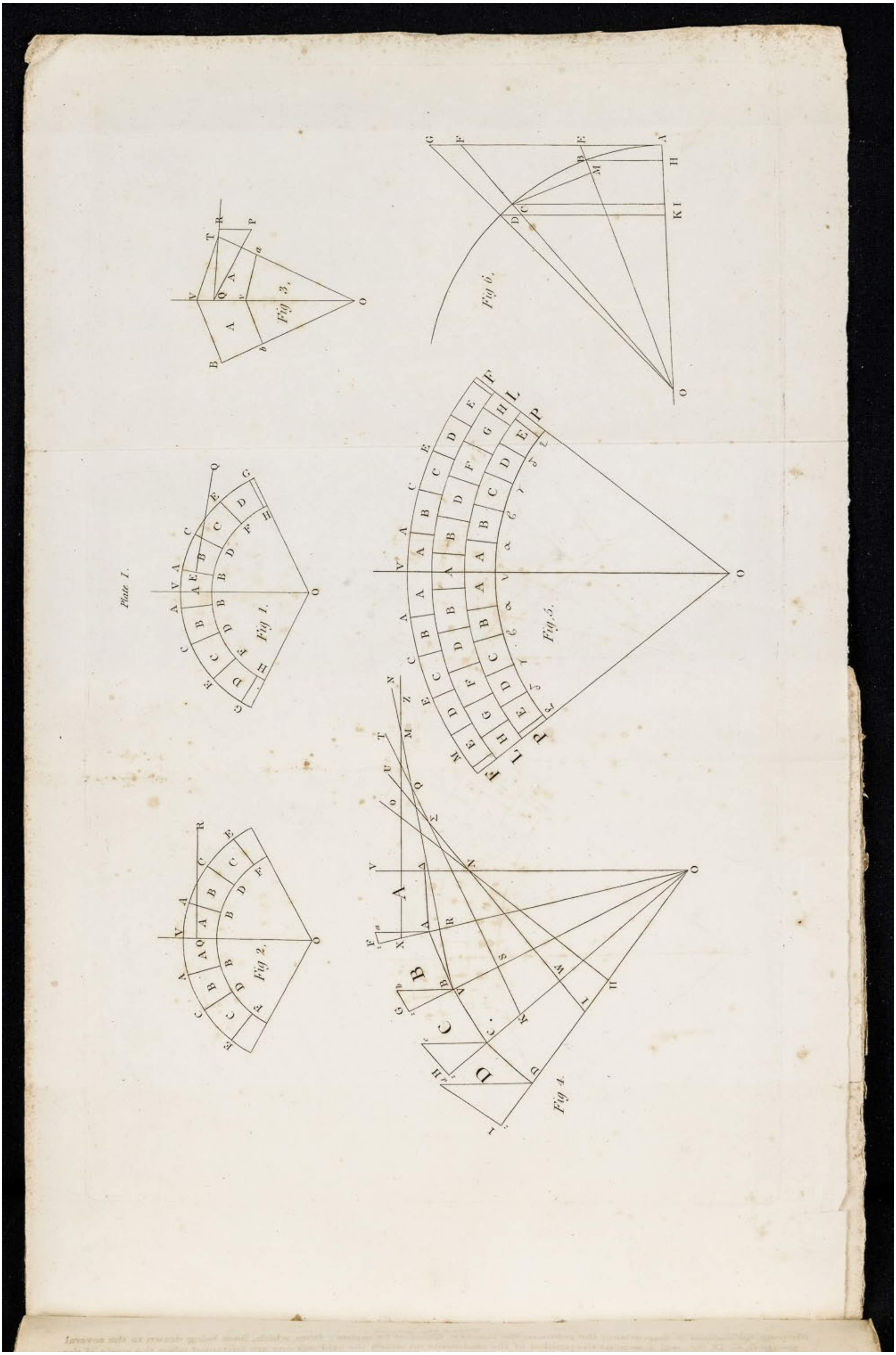
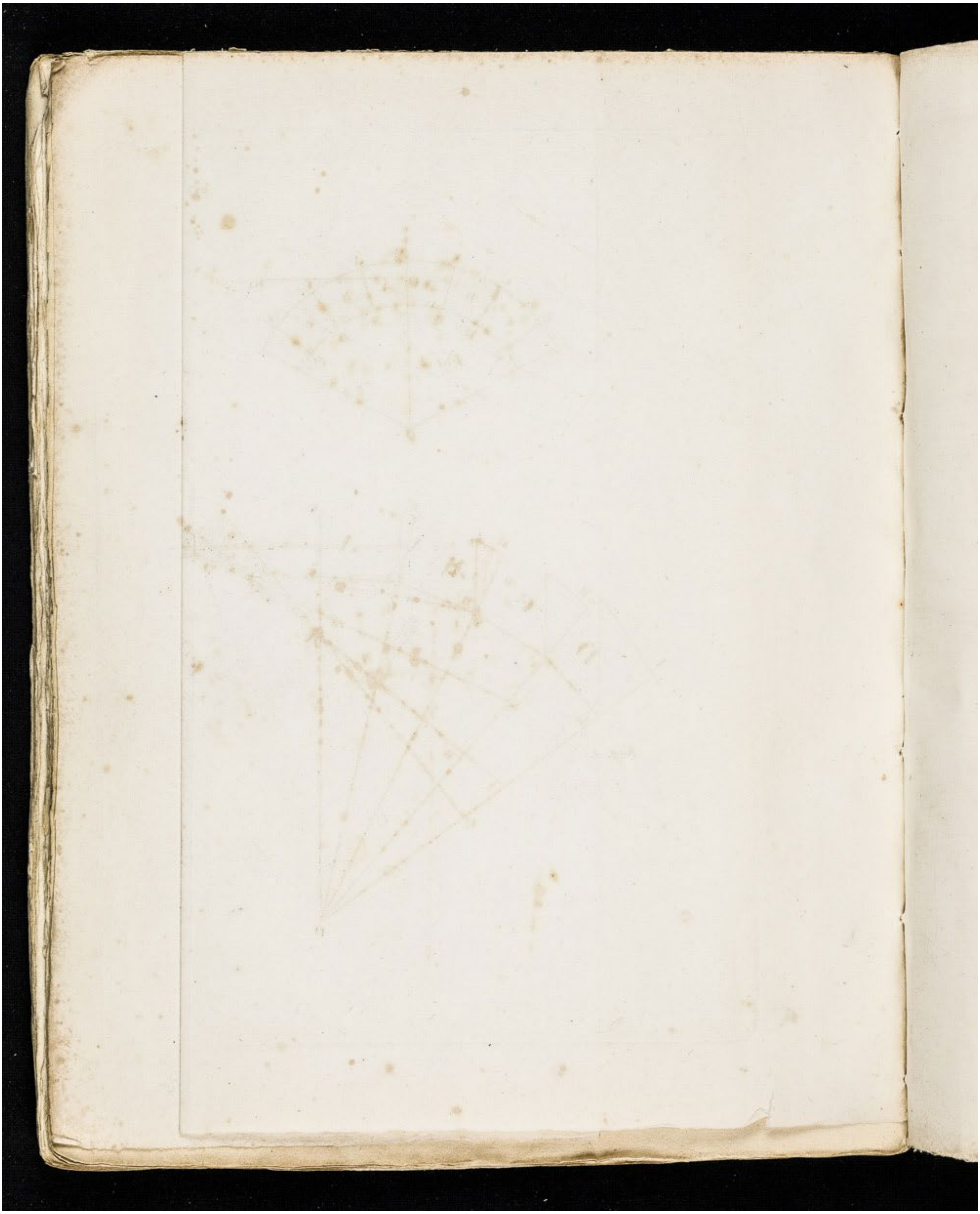


Plate I.
Fig 1.
Fig 2.
Fig 3.
Fig 4.
Fig 5.
Fig 6.



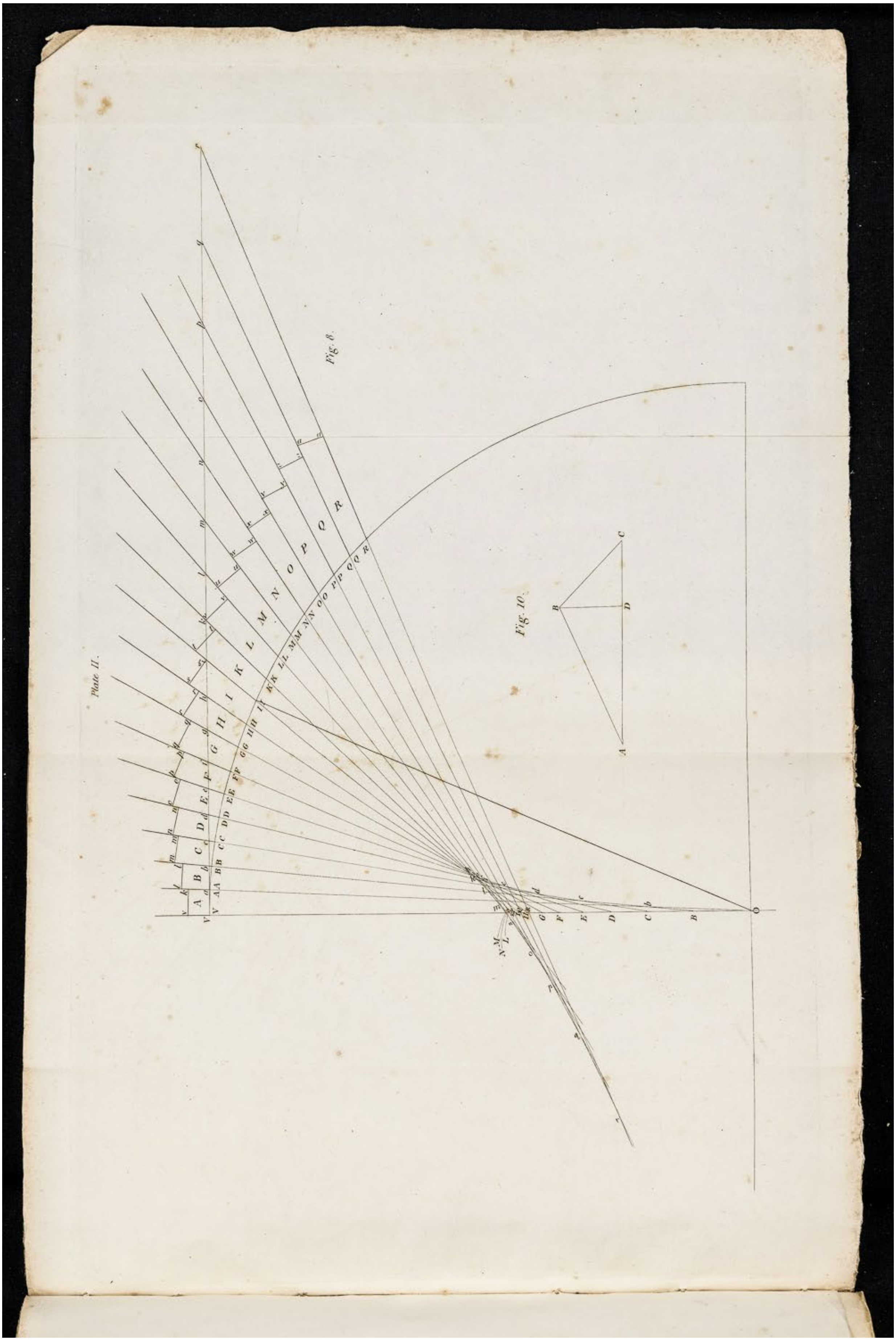
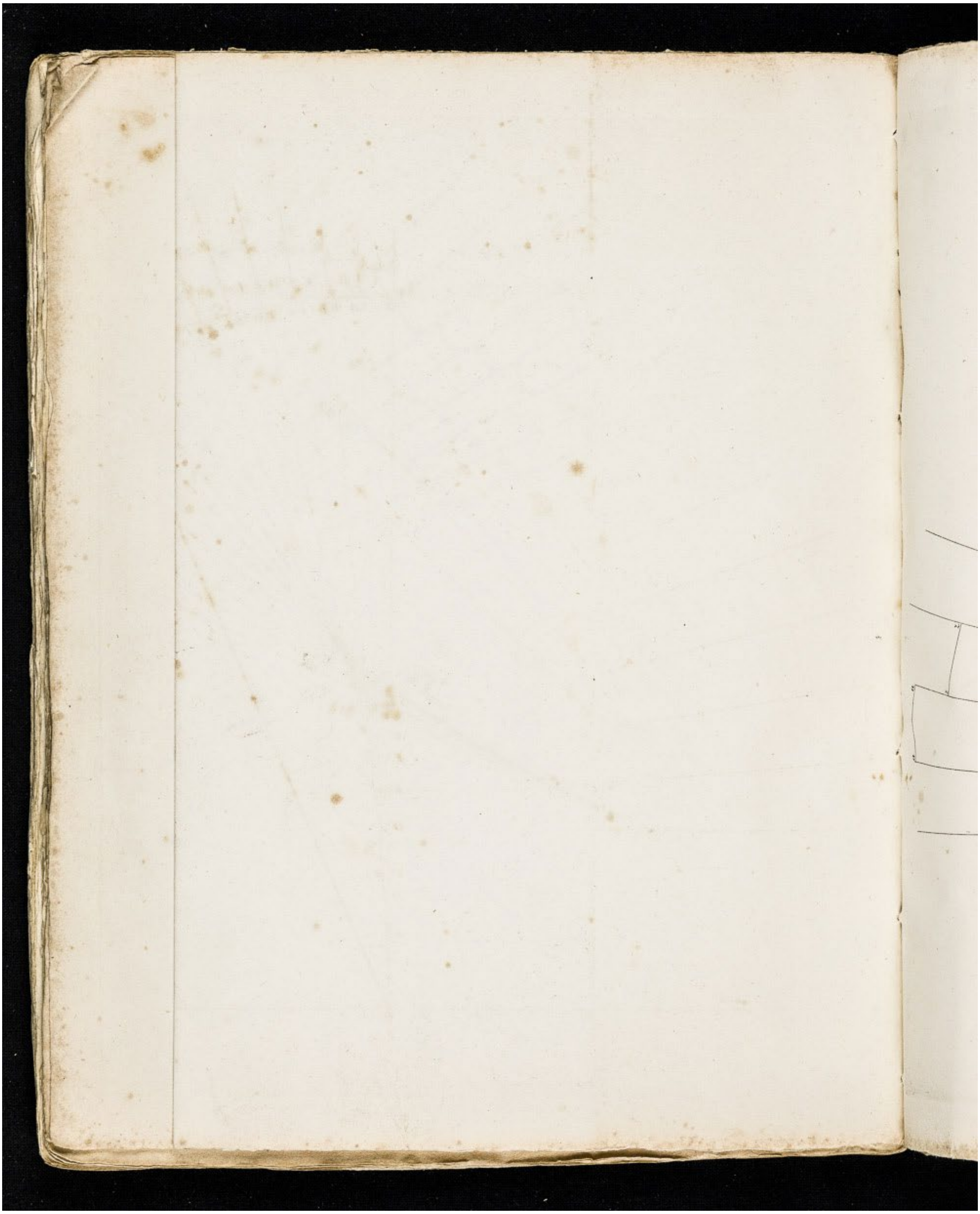


Plate II.
Fig. 8.
Fig. 10.



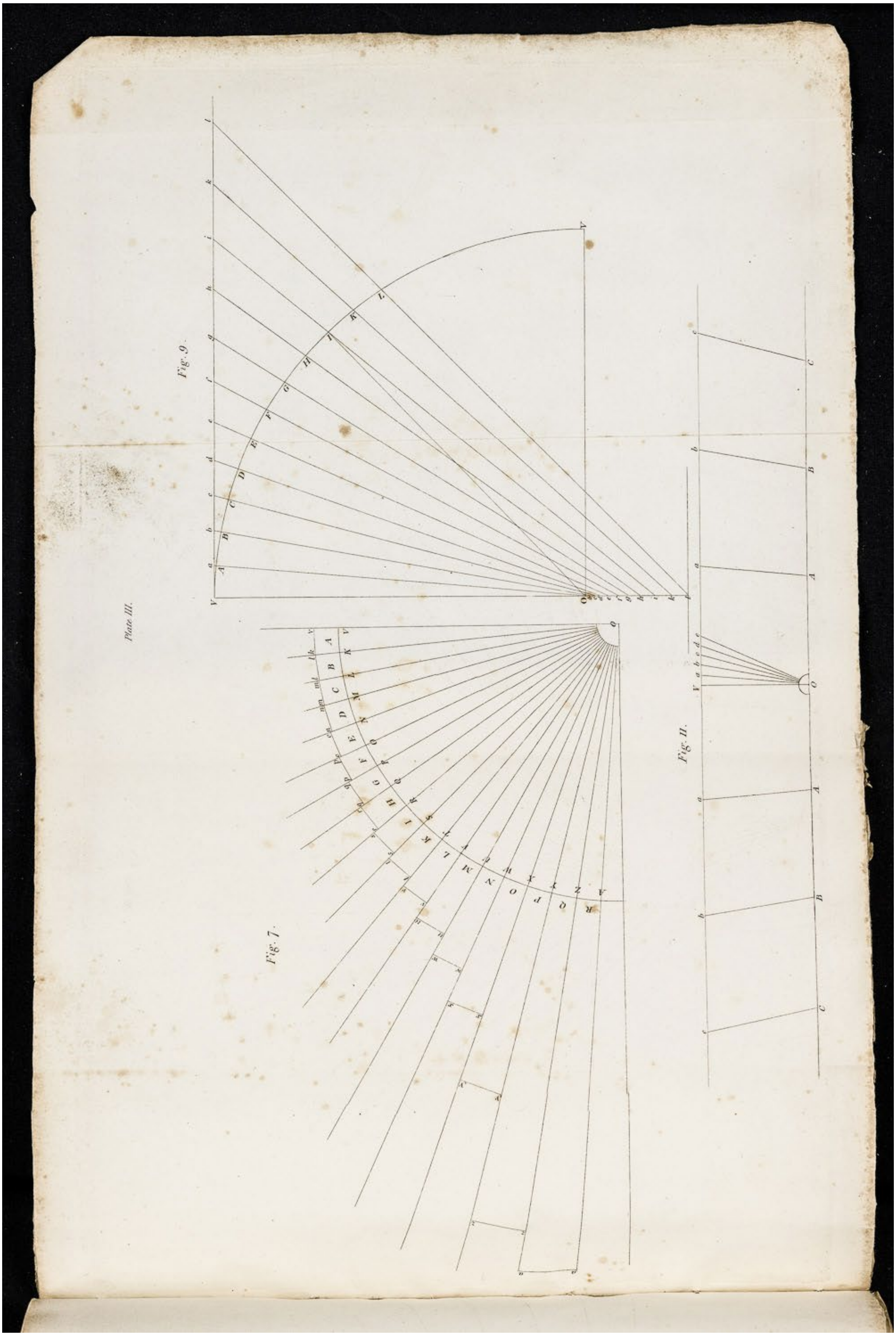


Plate III.
Fig. 9.
Fig. 7.
Fig. 11.

Plate IV.

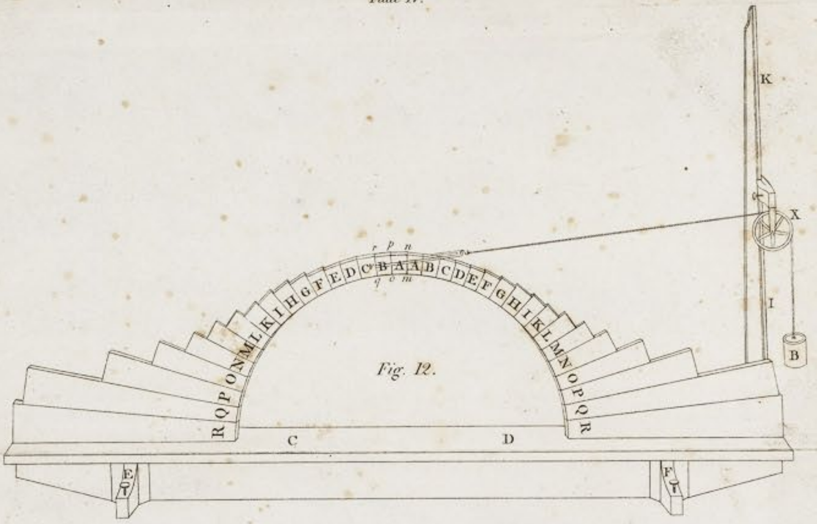


Fig. 12.

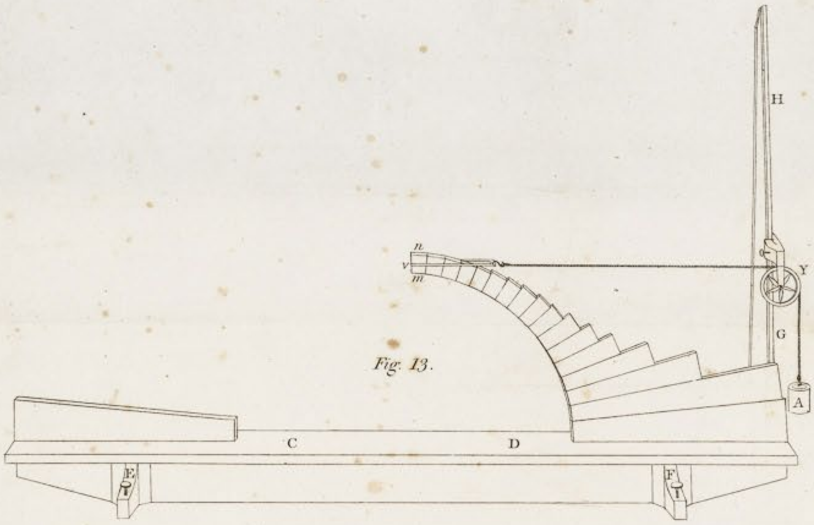


Fig. 13.

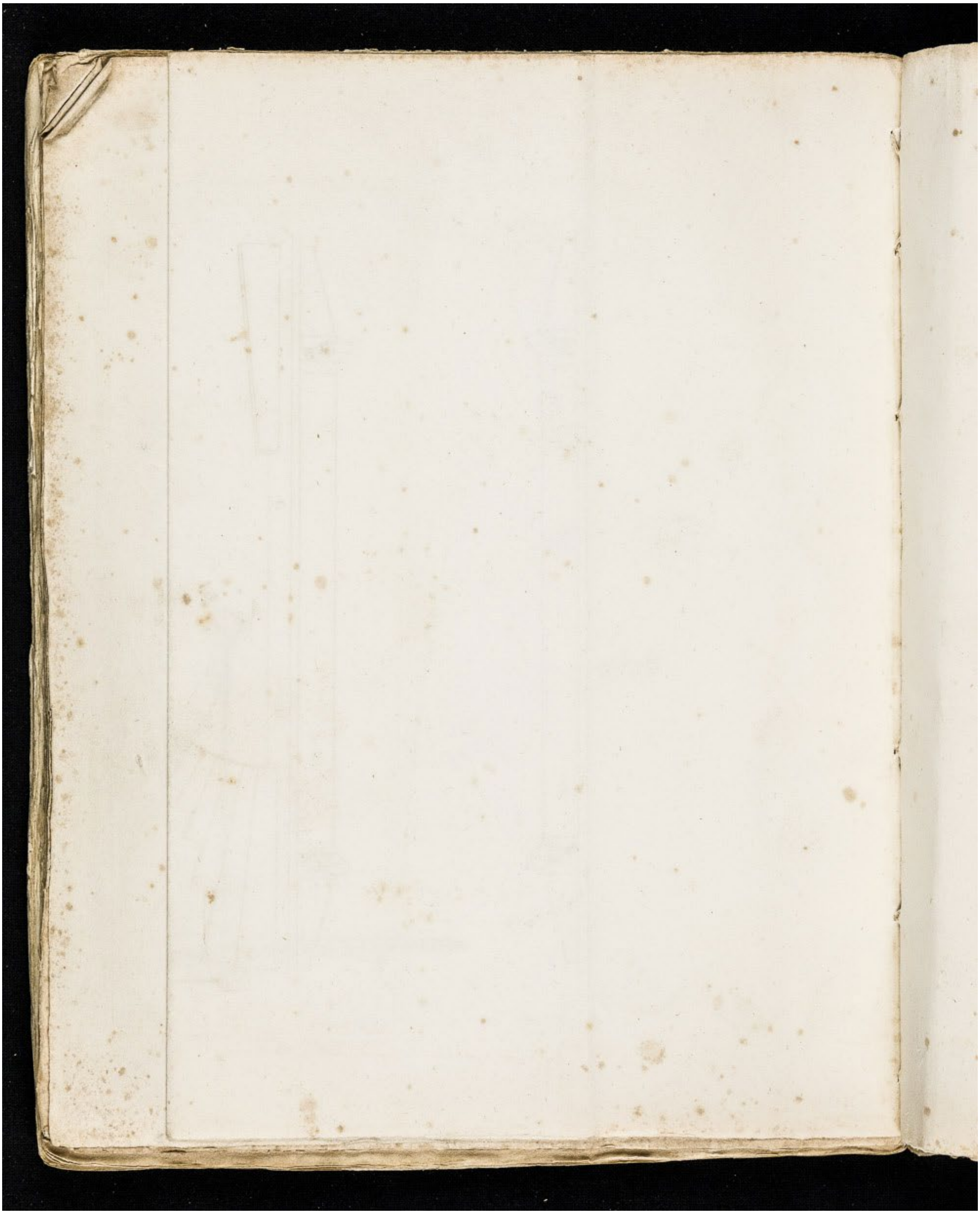


Plate. V.

Fig. 14.

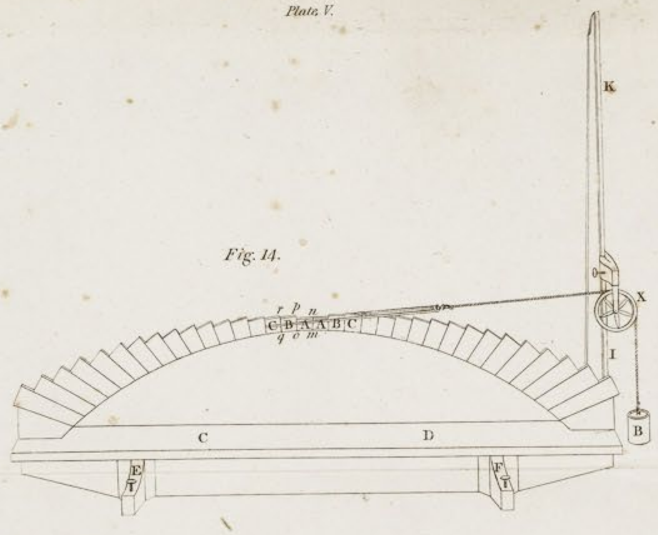


Fig. 15.